THE ONTARIO CURRICULUM

Mathematics

GRADE 9, DE-STREAMED (MTH1W)

2021

Ontario

The Ontario Public Service endeavours to demonstrate leadership with respect to accessibility in Ontario. Our goal is to ensure that Ontario government services, products, and facilities are accessible to all our employees and to all members of the public we serve. This document, or the information that it contains, is available, on request, in alternative formats. Please forward all requests for alternative formats to ServiceOntario at 1-800-668-9938 (TTY: 1-800-268-7095).

ISBN 978-1-4868-6445-4 (PDF)
ISBN 978-1-4868-6444-7 (HTML)

Educators should be aware that, with the exception of **the Grade 9 Mathematics course, 2021 (MTH1W)**, the 2005 Mathematics curriculum for Grade 10 and the 2007 Mathematics curriculum for Grades 11–12 remain in effect. All secondary mathematics courses for Grades 10–12 will continue to be based on those documents. All references to Grade 9 that appear in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* and *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007* have been superseded by *The Ontario Curriculum, Grade 9: Mathematics, 2021*. Addenda have been issued to the Grade 10 MPM2D and MFM2P courses, to be implemented for the 2022–23 school year.

Version history:

Version Date	**Description**
June 9, 2021	New Grade 9 Mathematics, De-streamed course (MTH1W) issued. This course replaces the Grade 9 Principles of Mathematics, Academic course (MPM1D); the Grade 9 Foundations of Mathematics, Applied course (MFM1P); and the Mathematics Transfer Course, Grade 9, Applied to Academic (MPM1H).
July 13, 2021	Edits to the Introduction, Elements of the Grade 9 Mathematics Course, and Cross-Curricular and Integrated Learning in Mathematics sections of Grade 9 Mathematics (MTH1W)
April 1, 2022	Examples and instructional tips added to Grade 9 Mathematics (MTH1W)
May 3, 2022	Teacher prompts and sample tasks added to Grade 9 Mathematics (MTH1W)

Contents

Une publication équivalente est disponible en français sous le titre suivant : *Mathématiques 9^{e} année, cours décloisonné, MTH1W* (2021).

Introduction

Preface

This curriculum policy presents the compulsory Grade 9 mathematics course, 2021 (MTH1W). This course supersedes the two Grade 9 courses outlined in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* as well as *The Ontario Curriculum: Mathematics – Mathematics Transfer Course, Grade 9, Applied to Academic, 2006.* Effective September 2021, all mathematics programs for Grade 9 will be based on the expectations outlined on this site.

The Grade 9 mathematics curriculum focuses on key mathematics concepts and skills, as well as on making connections between related math concepts, between mathematics and other disciplines, and between mathematics and the lived experiences of students. This curriculum is designed to support all students in developing an understanding of, and the ability to apply, the range of mathematical knowledge and skills appropriate for the grade level. Consequently, this curriculum is intended to support all students in continuing to build confidence in approaching mathematics, develop a positive attitude towards mathematics, think critically, work collaboratively, and feel that they are reflected in mathematics learning.

Vision and Goals of the Grade 9 Mathematics Course

The needs of learners are diverse, and all learners have the capacity to develop the knowledge, concepts, skills, and perspectives they need to become informed, productive, and responsible citizens in their own communities and within the world.

How mathematics is contextualized, positioned, promoted, discussed, taught, learned, evaluated, and applied affects the learning experiences and academic outcomes of all students. Mathematics can be appreciated for its innate beauty, as well as for its role in making sense of the world. Having a solid foundation in, a deep appreciation for, and excitement about mathematics, as well as recognizing their identities, lived experiences, and communities in their mathematics learning, will help ensure that all students grow more confident and capable as they step into the future.

All students bring their mathematical experiences from various contexts to school. Educators can value and build on these lived experiences so that mathematics classrooms become spaces that honour diverse mathematical ideas and thoughts, and incorporate multiple ways of knowing and doing. Such spaces allow all students to become flexible and adaptive learners in an ever-changing world.

The vision of this mathematics course is to support all students as they develop healthy and strong identities as mathematics learners and grow to be mathematically skilled, to enhance their ability to use mathematics to make sense of the world around them, and to enable them to make critical decisions while engaged in mathematical thinking. This vision is attained in a mathematics classroom filled with high academic expectations and deep engagement that generates enthusiasm and curiosity – an inclusive classroom where all students receive the highest-quality mathematics instruction and learning opportunities, are empowered to interact as confident mathematics learners, and are thereby supported in reaching their full potential.

The goal of the Ontario mathematics curriculum is to provide all students with the key skills required to:

- understand the importance of and appreciate the beauty and wonder of mathematics;
- recognize and appreciate multiple mathematical perspectives;
- make informed decisions and contribute fully to their own lives and to today's interconnected local and global communities;
- adapt to changes and synthesize new ideas;
- work both independently and collaboratively to approach challenges;
- communicate effectively;
- think critically and creatively to connect, apply, and leverage mathematics within other areas of study including science, technology, engineering, the arts, and beyond.

A strong foundation of mathematics is an important contributor to students' future success and an essential part of becoming an informed citizen. In order to develop a strong understanding of mathematics and the ability to apply mathematics in real life, all students must feel that they are connected to the curriculum – to what is taught, why it is taught, and how it is taught.

The Importance and Beauty of Mathematics

Mathematics is integral to every aspect of daily life – social, economic, cultural, and environmental. It is embedded into the rich and complex story of human history. People around the world have used, and continue to contribute, mathematical knowledge, skills, and attitudes to make sense of the world around them and to develop new mathematical thinking and appreciation for mathematics. Mathematics is conceptualized and practised in many different ways across diverse local and global cultural contexts. It is part of diverse knowledge systems composed of culturally situated thinking and practices. From counting systems, measurement, and calculation to geometry, spatial sense, trigonometry, algebra, functions, calculus, and statistics, mathematics has been evident in the daily lives of people and communities across human histories.

Today, mathematics is found all around us. For example, mathematics can be found in sports performance analysis, navigation systems, electronic music production, computer gaming, graphic art, quantum physics, climate change modelling, and so much more. Mathematics skills are necessary when we buy goods and services online, complete our taxes, do beading, construct buildings, and play sports. Mathematics also exists in nature, storytelling, music, dancing, puzzles, and games. Proficiency with mathematical ideas is needed for many careers, including but not limited to engineering, health care and medicine, psychology, computer science, finance, landscape design, fashion design, architecture, agriculture, ecology, the arts, the culinary arts, and many other skilled trades. In fact, in every field of pursuit, the analytical, problem-solving, critical-thinking, and creative-thinking skills that students develop through the study of mathematics are evident. In the modern age of evolving technologies, artificial intelligence, and access to vast sources of information and big data, knowing how to navigate, interpret, analyse, reason, evaluate, and problem solve is foundational to everyday life.

Mathematics can be understood as a way of studying and understanding structure, order, patterns, and relationships. The power of mathematics is evident in the connections among seemingly abstract mathematical ideas. The applications of mathematics have often yielded fascinating representations and results. As well, the aesthetics of mathematics have also motivated the development of new mathematical thinking. The beauty in mathematics can be found in the process of deriving elegant and succinct approaches to resolving problems.

At times, messy problems and seeming chaos may culminate in beautiful, sometimes surprising, results that are both simple and generalizable. Elegance and chaos are both integral to the beauty of mathematics itself and to the mathematical experience. In other words, the beauty of mathematics is illustrated and enhanced by students' diverse interpretations, strategies, representations, and identities – not diminished by them. Most importantly, students can experience wonder and beauty when they make exciting breakthroughs in problem solving. Therefore, these two aspects of mathematics, aesthetics and application, are deeply interconnected.

The Grade 9 mathematics course strives to equip all students with the knowledge, skills, and habits of mind that are essential to understanding and enjoying the importance and beauty of mathematics. Learning in Grade 9 mathematics begins with a focus on the fundamental concepts and foundational skills. This leads to an understanding of mathematical structures, operations, processes, and language that provides students with the means necessary for reasoning, justifying conclusions, and expressing and communicating mathematical ideas.

When educators put student learning at the centre, provide relevant and meaningful learning opportunities, and use technology strategically to enhance learning experiences, all students are supported as they learn and apply mathematical concepts and skills within and across strands and other subject areas.

The Grade 9 mathematics course emphasizes the importance of establishing an inclusive mathematical learning community where all students are invited to experience the living practice of mathematics, to work through challenges, and to find beauty and success in problem solving. As students engage with the curriculum, they are supported in incorporating their lived experiences and existing mathematical understandings, and then integrating the new ideas they learn into their daily lives. When students recognize themselves in what is taught and how it is taught, they begin to view themselves as competent and confident mathematics learners who belong to the larger mathematics community. As students develop mathematical knowledge and skills, they grow as mathematical thinkers. As students explore histories of mathematics and comprehend the importance and beauty of mathematics, they develop their mathematical agency and identity, at the same time as they make connections to other subjects and the world around them.

Human Rights, Equity, and Inclusive Education in Mathematics

Research indicates that there are groups of students (for example, Indigenous students, Black students, students experiencing homelessness, students living in poverty, students with LGBTQ+ identities, and students with special education needs and disabilities) who continue to experience systemic barriers to accessing high-level instruction in and support with learning mathematics. Systemic barriers, such as racism, implicit bias, and other forms of discrimination, can result in inequitable academic and life outcomes, such as low confidence in one's ability to learn mathematics, reduced rates of credit completion, and leaving the secondary school system prior to earning a diploma. Achieving equitable outcomes in mathematics for all students requires educators to be aware of and identify these barriers, as well as the ways in which they can overlap and intersect, which can compound their effect on student well-being, student success, and students' experiences in the classroom and in the school. Educators must not only know about these barriers, they must work actively and with urgency to address and remove them.

Students bring abundant cultural knowledges, experiences, and competencies into mathematical learning. It is essential for educators to develop pedagogical practices that value and centre students' prior learning, experiences, strengths, and interests. Such pedagogical practices are informed by and build on students' identities, lived experiences, and linguistic resources. When educators employ such pedagogy, they hold appropriate and high academic expectations of students, applying the principles of Universal Design for Learning and differentiated instruction to provide multiple entry points and maximize opportunities for all students to learn. By acknowledging and actively working to eliminate the systemic barriers that some students face, educators create the conditions for authentic experiences that empower student voices and enhance their sense of belonging, so that each student can develop a healthy identity as a mathematics learner and can succeed in mathematics and in all other subjects. Mathematics learning that is student-centred allows students to find relevance and meaning in what they are learning and to make connections between the curriculum and the world outside the classroom.

In mathematics classrooms, teachers also provide opportunities for cross-curricular learning and for teaching about human rights. To create anti-racist, anti-discriminatory learning environments, all educators must be committed to equity and inclusion and to upholding and promoting the human rights of every learner. Students of all identities and social locations have the right to mathematics opportunities that allow them to succeed, personally and academically. In any mathematics classroom, it is crucial to acknowledge students' intersecting social identities and their connected lived realities. Educators have an obligation to develop and nurture learning environments that are reflective of and responsive to students' strengths, needs, cultures, and diverse lived experiences – identity-affirming learning environments free from discrimination. In such learning environments, educators set appropriate and high academic expectations for all.

Culturally Responsive and Relevant Pedagogy in Mathematics

High-quality instruction that emphasizes deep mathematical thinking and cultural and linguistic knowledge and that addresses issues of inequity is the foundation of culturally responsive and relevant pedagogy (CRRP) in mathematics. In CRRP classrooms, teachers reflect on their own identities and pay attention to how those identities affect their teaching, their ideas, and their biases. Teachers also learn about students' identities, identifications, and/or affiliations and connected lived experiences. Teachers develop an understanding of how students are thinking about mathematical concepts according to their cultural backgrounds and experiences, and make connections with these cultural ways of knowing in their pedagogy. This approach to pedagogy develops social consciousness and critique while valorizing students' cultural backgrounds, communities, and cultural and linguistic competences. Teachers build on students' experiences, ideas, questions, and interests to support the development of an engaging and inclusive mathematics classroom community.

In mathematics classrooms, educators use CRRP to create teaching and learning opportunities to engage students in shaping much of the learning and to promote mathematical agency investment in the learning. When students develop agency, they are motivated to take ownership of their learning of, and progress in, mathematics. Teaching about diverse mathematical approaches and figures in history, from different global contexts, can offer opportunities for students to feel that they are reflected in mathematical learning – a key factor in developing students' sense of self – and to learn about others, and about the multiple ways mathematics exists in all aspects of the world around them.

Mathematics is situated and produced within cultures and cultural contexts. The curriculum is intended to expand historical understanding of the diversity of mathematical thought. In an anti-racist and anti-discriminatory environment, teachers know that there is more than one way to develop a solution, and students are exposed to multiple ways of knowing and encouraged to explore multiple ways of finding answers.

Indigenous pedagogical approaches emphasize holistic, experiential learning, teacher modelling, and the use of collaborative and engaging activities. Teachers differentiate instruction and assessment opportunities to encourage different ways of learning, to allow students to learn from and with each other, and to promote an awareness of and respect for the diverse and multiple ways of knowing that are relevant to and reflective of students' lived experiences in classrooms, schools, and the world. When making connections between mathematics and real-life applications, teachers are encouraged to work in partnership with First Nations, Inuit, and Métis individuals, communities, and/or nations. Teachers may respectfully incorporate culturally specific examples that highlight First Nations, Inuit, and Métis cultures, histories, present-day realities, ways of knowing, and contributions, to infuse Indigenous knowledges and perspectives meaningfully and authentically into the mathematics program. In this way, culturally specific examples centre Indigenous students as mathematical thinkers, and strengthen learning and course content so that all students continue to learn about diverse cultures and communities in a respectful and informed way. Students' mind, body, and spirit are nourished through connections and creativity.

More information on equity and inclusive education can be found in the “Human Rights, Equity, and Inclusive Education” subsection of “Considerations for Program Planning”.

Principles Underlying the Grade 9 Mathematics Curriculum

- **A mathematics curriculum is most effective when it values and honours the diversity that exists among students and within communities.**
  The Grade 9 mathematics curriculum is based on the belief that all students can and deserve to be successful in mathematics. In particular, an inclusive curriculum is built on the understanding that not all students necessarily learn mathematics in the same way, use the same resources (e.g., tools and materials), or learn within the same time frames. Setting high academic expectations and building a safe and inclusive community of learners requires the purposeful use of a variety of instructional and assessment strategies and approaches that build on students’ prior learning and experiences, and create an optimal and equitable environment for mathematics learning. The curriculum emphasizes the need to eliminate systemic barriers and to serve students belonging to groups that have been historically disadvantaged and underserved in mathematics education.
- **A robust mathematics curriculum is essential for ensuring that all students reach their full potential.**
  The Grade 9 mathematics curriculum challenges all students by including learning expectations that build on students’ prior knowledge and experience; involve higher-order thinking skills; and require students to make connections between their lived experiences, mathematical concepts, other subject areas, and situations outside of school. This learning enables all students to gain a powerful knowledge of the usefulness of the discipline and an appreciation of the histories and importance of mathematics.
- **A mathematics curriculum provides all students with the fundamental mathematics concepts and foundational skills they require to become capable and confident mathematics learners.**
  The Grade 9 mathematics curriculum provides a balanced approach to the teaching and learning of mathematics. It is based on the belief that all students learn mathematics most effectively when they can build on prior knowledge to develop a solid understanding of the concepts and skills in mathematics, and when they are given opportunities to apply these concepts and skills as they solve increasingly complex tasks and investigate mathematical ideas, applications, and situations in everyday contexts. As students continue to explore the relevance of mathematics, they further develop their identity and agency as capable mathematics learners.
- **A progressive mathematics curriculum includes the strategic integration of technology to support and enhance the learning and doing of mathematics.**
  The Grade 9 mathematics curriculum strategically integrates the use of appropriate technologies to support all students in developing conceptual understanding and procedural fluency, while recognizing the continuing importance of students’ mastering the fundamentals of mathematics. For some students, assistive technology also provides an essential means of accessing the mathematics curriculum and demonstrating their learning. Students develop the ability to select appropriate tools and strategies to perform particular tasks, to investigate ideas, and to solve problems. The curriculum sets out a framework for learning important skills, such as problem

solving, coding, and modelling, as well as opportunities to develop critical data literacy, information literacy, and financial literacy skills.

- **A mathematics curriculum acknowledges that the learning of mathematics is a dynamic, gradual, and continuous process, with each stage building on the last.**
  The Grade 9 mathematics curriculum is dynamic, continuous, and coherent and is designed to support all students in developing an understanding of the interconnected nature of mathematics. Students come to understand how concepts develop and how they build on one another. As students communicate their reasoning and findings, they move towards new understandings. Teachers observe and listen to all students and then responsively shape instruction in ways that foster and deepen student understanding of important mathematics. The fundamental concepts, skills, and processes introduced in the elementary grades support students in extending their learning in the secondary grades.
- **A mathematics curriculum is integrated with the world beyond the classroom.**
  The Grade 9 mathematics curriculum provides opportunities for all students to investigate and experience mathematical situations they might find outside the classroom and develop an appreciation for the beauty and wide-reaching nature and importance of mathematics. The overall curriculum integrates and balances concept development and skill development, including social-emotional learning skills, as well as the use of mathematical processes and real-life applications.
- **A mathematics curriculum motivates students to learn and to become lifelong learners.**
  The Grade 9 mathematics curriculum is brought to life in the classroom, where students develop mathematical understanding and are given opportunities to connect their knowledge and skills to wider contexts and other disciplines. Making connections to the world around them stimulates their interest and motivates them to become lifelong learners with healthy attitudes towards mathematics. Teachers bring the mathematics curriculum to life using their knowledge of:
  - the mathematics curriculum;
  - the backgrounds and identities of all students, including their past and ongoing experiences with mathematics and their learning strengths and needs;
  - mathematical concepts and skills, and the ways in which they are connected across the strands, other grades, other disciplines, and the world outside the classroom;
  - instructional approaches and assessment strategies best suited to meet the learning needs of each student;
  - resources designed to support and enhance the achievement of and engagement with the curriculum expectations, while fostering an appreciation for and joy in mathematics learning.

Roles and Responsibilities

Students

It is essential that all students continue to develop a sense of responsibility for and ownership of their own learning as they begin their journey through secondary school. Mastering the skills and concepts connected with learning in the mathematics curriculum requires a commitment to:

- continual and consistent personal reflection and goal setting;
- a belief that they are capable of succeeding in mathematics;
- developing the skills to persevere when taking on new challenges;
- connecting prior experiences, knowledge, skills, and habits of mind to new learning;
- a willingness to work both independently and collaboratively in an inclusive environment;
- dedication to ongoing practice;
- a willingness and an ability to receive and respond to meaningful feedback and ask questions to clarify understanding;
- a willingness to explore new learning in mathematics and share insights and experiences.

Through ongoing practice and reflection, all students can develop a strong and healthy mathematical identity whereby they value and appreciate mathematics as a discipline, feel themselves to be confident and competent mathematics learners, and understand what successful mathematics learning and being an effective mathematician look like.

Students' experiences influence their attitudes towards mathematics education and can have a significant impact on their engagement with mathematics learning and their subsequent success in achieving the expectations. Students who are engaged in their learning and who have opportunities to solve interesting, relevant, and meaningful problems within a supportive and inclusive learning environment are more likely to adopt practices and behaviours that support mathematical thinking. More importantly, they are more likely to be successful in their learning, which contributes to their enjoyment of mathematics and increases their desire to pursue further mathematics learning.

With teacher support and encouragement, students learn that they can apply the skills they acquire in mathematics to other contexts and subjects. For example, they can apply the problem-solving skills they develop in mathematics to their study of the science and Canadian and world studies curricula. They can also make connections between their learning and life beyond the classroom. For example, when presented with an issue or a contextually relevant STEM-based (science, technology, engineering, and mathematics–based) problem, they can look for potential applications of mathematical modelling. They can also begin to identify how mathematical modelling can be used to answer important questions related to global health, the environment, and sustainable, innovative development, or to address various issues that are relevant to their lives and communities.

Parents

Parents[1] are significant role models for their children and play an integral part in their children's experiences with mathematics. It is important for schools and parents, and in some situations, caring and trusted adults in students' lives who are not their parents, to work together to ensure that they provide a mutually supportive framework for young people's mathematics education. Research assures us of the positive impact of parent engagement and parent-child communication about mathematics on student success.

Parents can play a role in their children's success by speaking positively about mathematics and modelling the attitude that mathematics is enjoyable, worthwhile, and valuable. By encouraging their children to acknowledge challenges, to persevere when solving problems, and to believe that they can succeed in mathematics, parents help them build self-confidence and a sense of identity as mathematics learners.

Parents can support their children's mathematics success by showing an interest in what their children are learning. Parents are encouraged to engage with mathematics alongside their children by asking about their experiences in class and by finding ways to apply what is being learned in class to everyday contexts. Mathematics is everywhere, and parents can help their children make connections between what they are learning at school and everyday experiences at home and in the community, using tasks such as making appropriate choices when shopping, or saving for future needs. Parents can include their children in the things they do themselves that involve mathematics, such as estimating the amount of material needed to redecorate or renovate a room, or the quantities of ingredients needed to cook a meal. Through family activities, such as enjoying mathematics-based puzzles and games, making crafts, and beading jewelry together, parents can create opportunities for mental mathematics estimations and calculations and for making predictions. Parents can support their children's learning by encouraging them to complete their mathematics tasks, to practice new skills and concepts, to apply new mathematics learning to experiences at home, and to connect mathematical experiences at home to learning at school.

As students begin their journey through secondary school, parents can help them consider how mathematics may play a role in their future by talking about education and career goals or connecting with community partners to gather information. Parents can help their children make connections between what they are learning, potential careers, and their future choice of postsecondary pathways – such as apprenticeship, skilled trades, community living, college, university, or the workplace.

Schools offer a variety of opportunities for parents to learn more about how to support their children's mathematics learning: for example, events related to mathematics may be held at the school; teachers

---

[1] The word *parent(s)* is used on this website to refer to parent(s) and guardian(s). It may also be taken to include caregivers or close family members who are responsible for raising the child.

may provide newsletters or communicate with parents through apps or social media; and school or board websites may provide helpful tips about how parents can engage in their children's mathematics learning outside of school and may even provide links where they can learn more or enjoy mathematics-related activities together.

If parents need more information about what their children are learning, and how to support their children's success in mathematics, teachers are available to answer questions and provide information and resources.

Teachers

Teachers have the most important role in the success of students in mathematics. Teachers are responsible for ensuring that all students receive the highest quality of mathematics education. This requires them to have high academic expectations of all students, provide appropriate supports for learning, and believe that all students are capable math learners. Teachers bring expertise and skills to providing varied and equitable instructional and assessment approaches to the classroom. Teachers plan a mathematics program using an asset-based approach that affirms students' identities, reflects their lived experiences, leverages their strengths, and addresses their needs in order to ensure equitable, accessible, and engaging learning opportunities for every student. The attitude with which teachers themselves approach mathematics is critical, as teachers are important role models for students.

Teachers place students' well-being and academic success at the centre of their mathematics planning, teaching, and assessment practices, and understand how the learning experiences they provide will develop an appreciation of mathematics and foster a healthy attitude and engagement in all students. Teachers have a thorough understanding of the mathematics content they teach, which enables them to provide relevant and responsive, high-quality mathematical opportunities through which all students can develop their understanding of mathematical knowledge, concepts, and skills. Teachers understand the learning continua along which students develop their mathematical thinking and, with effective use of direct instruction and high-quality mathematical tasks, can thus support all students' movement along these continua. Teachers provide ongoing meaningful feedback to all students about their mathematics learning and achievement, which helps to build confidence and provide focused next steps. Teachers support students in developing their ability to solve problems, reason mathematically, and connect the mathematics they are learning to the real world around them. They recognize the importance of emphasizing and illustrating the usefulness of mathematics in students' lives, and of integrating mathematics with other areas of the curriculum – such as making connections with science, engineering, art, and technology to answer scientific questions or solve problems, or engaging in political debate and community development. They recognize the importance of supporting students in learning about careers involving mathematics, and of supporting the development of students' mathematical agency to grow their identity as capable mathematical thinkers.

As part of effective teaching practice, teachers use multiple ways and both formal and informal means to communicate with parents and develop partnerships between home or caring adults and school that meet the varied needs of families. Through various types of communication, teachers discuss with parents or caring adults what their children are learning in mathematics at school. These communications also help teachers better understand students' mathematical experiences beyond the classroom, and learn more about students' interests, skills, and aspirations. Ongoing communication leads to stronger connections between the home, community, and school to support student learning and achievement in mathematics.

Principals

Principals model the importance of lifelong learning and understand that mathematics plays a vital role in the future success of students. Principals provide instructional leadership for the successful implementation of the mathematics curriculum – in the school and in communications with parents – by emphasizing the importance of a well-planned mathematics program and high-quality mathematical instruction, by promoting the idea that all students are capable of becoming confident mathematics learners, and by encouraging a positive and proactive attitude towards mathematics and student agency in mathematics.

Principals work in partnership with teachers and parents to ensure that all students have access to the best possible educational experience. To support student learning, principals monitor the implementation of the Ontario mathematics curriculum. Principals ensure that English language learners are being provided the accommodations and/or modifications they require for success in the mathematics program. Principals are also responsible for ensuring that every student who has an Individual Education Plan (IEP) is receiving the modifications and/or accommodations described in their plan – in other words, for ensuring that the IEP is properly developed, implemented, and monitored.

Ensuring that teachers have the competence, agency, support, confidence, resources, and tools they need to deliver a high-quality program is essential. Principals collaborate with teachers and school and system leaders to develop professional learning opportunities that deepen teachers' curriculum knowledge, mathematical content knowledge for teaching, and pedagogy, and enhance their self-efficacy in teaching mathematics.

Community Partners

Community partners are an important resource for a school's mathematics education program. Community partners can also contribute to the success of the program by providing support for families, children and youth, and educators, so that they in turn may support student learning. Relationships with local businesses, volunteer groups, Indigenous communities, postsecondary institutions, informal learning spaces such as museums and science centres, and community organizations such as those that serve newcomer families or marginalized communities, can provide opportunities for authentic perspectives and real-world application of mathematics, as well as support for families. Nurturing partnerships with other schools can facilitate the sharing of resources, strategies, and facilities, the development of professional learning opportunities for staff, and the hosting of special events such as mathematics or coding workshops for students.

Communities provide social contexts for learning, such as opportunities for volunteer work or employment for students at the secondary level. Students bring knowledge and experiences from their homes and communities that are powerful resources in creating productive learning environments. By involving members of the community, teachers and principals can position mathematics learning as collaborative and experiential. Membership in a community also supports students in developing a sense of belonging and in building their identity as mathematics learners in relation to, and with, others.

Elements of the Grade 9 Mathematics Course

Overview

The Grade 9 mathematics course builds on the elementary program and is based on the same fundamental principles.

The overall aim of the Grade 9 mathematics course is to ensure that all students can access any secondary mathematics course they need in order to pursue future studies and careers that are of interest to them.

This course is designed to be inclusive of all students in order to facilitate their transition from the elementary grades to the secondary level. It offers opportunities for all students to build a solid foundation in mathematics, broaden their knowledge and skills, and develop their mathematical identity. This approach allows students to make informed decisions in choosing future mathematics courses based on their interests, and in support of future plans for apprenticeship training, university, college, community living, or the workplace.

Similar to the elementary curriculum, the Grade 9 course adopts a strong focus on the processes that best enable students to understand mathematical concepts and learn related skills. Attention to the mathematical processes is considered essential to a balanced mathematics program. The seven mathematical processes identified in the curriculum include *problem solving*, *reasoning and proving*, *reflecting*, *connecting*, *communicating*, *representing*, and *selecting tools and strategies*.

Throughout the course, students actively participate in the learning of mathematics by making connections to their lived experiences and to real-life applications. They continue to develop critical consciousness of how socio-cultural structures within systems impact individual experiences and opportunities, and to shape their identities as mathematics learners.

Teachers implement the curriculum through effective assessment and instructional practices that are rooted in Culturally Responsive and Relevant Pedagogy. Teachers utilize a variety of assessment and instructional approaches that provide students with multiple entry points to access mathematics learning and multiple opportunities to demonstrate their achievement in mathematics.

This course continues the learning from Grade 8 and prepares students for success in all senior secondary mathematics courses in all pathways moving forward. Students who successfully complete the Grade 9 mathematics course may proceed to a mathematics course in Grade 10.

The following section is in effect for the 2021–22 school year and will be updated as the secondary mathematics program is revised. The 2005 Mathematics curriculum for Grade 10 and the 2007 Mathematics curriculum for Grades 11–12 remain in effect. All references to Grade 9 that appear in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* and *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007* have been superseded by the section below.

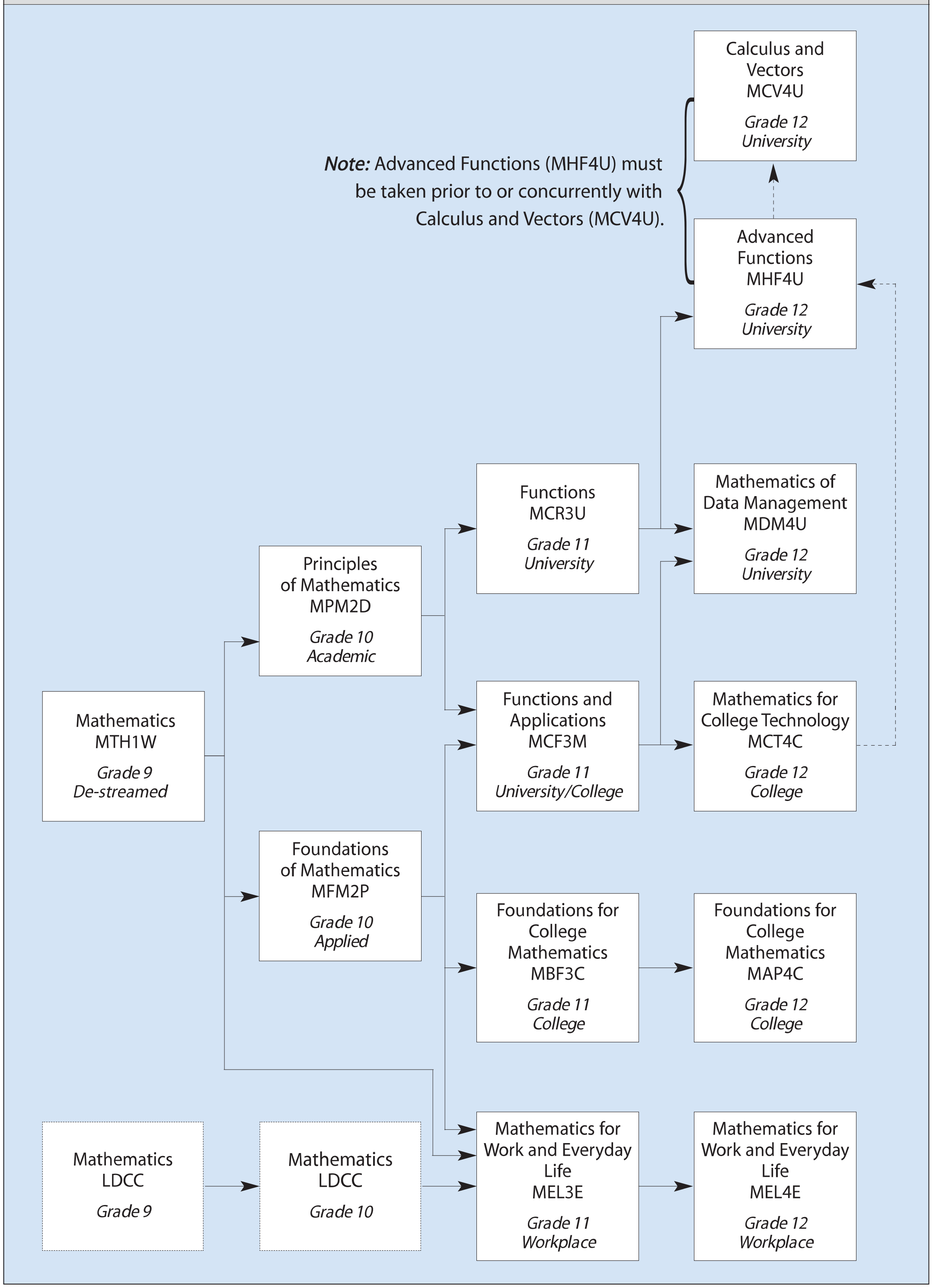
Courses in Mathematics, Grades 9 to 12

Grade	Course Name	Course Type	Course Code	Credit Value	Prerequisite
9	Mathematics	De-streamed	MTH1W	1.0	None
10	Principles of Mathematics	Academic	MPM2D	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Principles of Mathematics, Academic (2005)
10	Foundations of Mathematics	Applied	MFM2P	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Foundations of Mathematics, Applied (2005)
11	Functions	University	MCR3U	1.0	Grade 10 Principles of Mathematics, Academic
11	Functions and Applications	University/ College	MCF3M	1.0	Grade 10 Principles of Mathematics, Academic, or Grade 10 Foundations of Mathematics, Applied
11	Foundations for College Mathematics	College	MBF3C	1.0	Grade 10 Foundations of Mathematics, Applied
11	Mathematics for Work and Everyday Life	Workplace	MEL3E	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Principles of Mathematics, Academic (2005), or Grade 9 Foundations of Mathematics, Applied (2005), or a Grade 10 Mathematics LDCC (locally developed compulsory credit) course

12	Advanced Functions	University	MHF4U	1.0	Grade 11 Functions, University, or Grade 12 Mathematics for College Technology, College
12	Calculus and Vectors	University	MCV4U	1.0	Grade 12 Advanced Functions, University, must be taken prior to or concurrently with Calculus and Vectors
12	Mathematics of Data Management	University	MDM4U	1.0	Grade 11 Functions, University, or Grade 11 Functions and Applications, University/College
12	Mathematics for College Technology	College	MCT4C	1.0	Grade 11 Functions and Applications, University/College, or Grade 11 Functions, University
12	Foundations for College Mathematics	College	MAP4C	1.0	Grade 11 Foundations for College Mathematics, College, or Grade 11 Functions and Applications, University/College
12	Mathematics for Work and Everyday Life	Workplace	MEL4E	1.0	Grade 11 Mathematics for Work and Everyday Life, Workplace

Prerequisite Chart for Mathematics, Grades 9–12

This chart maps out all the courses in the discipline and shows the links between courses and the possible prerequisites for them. It does not attempt to depict all possible movements from course to course.



Note: LDCC – locally developed compulsory credit course (LDCC courses are not outlined in this curriculum.)

Note: For students who completed any of the Grade 9 mathematics courses prior to September 2021, refer to the prerequisite chart on page 10 of *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007*.

Locally Developed Compulsory Credit Courses (LDCCs)

School boards may offer up to two locally developed compulsory credit courses in mathematics – a Grade 9 course and/or a Grade 10 course – that may be used to meet the compulsory credit requirement in mathematics for one or both of these grades. The locally developed Grade 9 and/or Grade 10 compulsory credit courses prepare students for success in the Grade 11 and Grade 12 workplace preparation courses.

Half-Credit Courses

The course outlined in this curriculum is designed to be offered as a full-credit course. However, it may also be delivered as two half-credit courses. Half-credit courses, which require a minimum of fifty-five hours of scheduled instructional time, must adhere to the following conditions:

- The two half-credit courses created from a full course must together contain all of the expectations of the full course.
- The expectations for each half-credit course must be divided in a manner that best enables students to achieve the required knowledge and skills in the allotted time.
- A course that is a prerequisite for another course in the secondary curriculum may be offered as two half-credit courses, but students must successfully complete both parts of the course to fulfil the prerequisite. (Students are not required to complete both parts unless the course is a prerequisite for another course they wish to take.)
- The title of each half-credit course must include the designation Part 1 or Part 2. A half credit (0.5) will be recorded in the credit-value column of both the report card and the Ontario Student Transcript.

Boards will ensure that all half-credit courses comply with the conditions described above, and will report all half-credit courses to the ministry annually in the School October Report.

Curriculum Expectations for the Grade 9 Mathematics Course

The expectations identified for this course describe the knowledge, concepts, and skills that students are expected to acquire, demonstrate, and apply in their class work and tasks, on tests, in demonstrations, and in various other activities on which their achievement is assessed and evaluated.

Mandatory learning is described in the overall and specific expectations of the curriculum.

Two sets of expectations – overall expectations and specific expectations – are listed for each *strand*, or broad area of the curriculum. The strands in this course are lettered AA and A through F.

The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of the course. The *specific expectations* describe the expected knowledge, concepts, and skills in greater detail. The specific expectations are grouped under numbered subheadings, each of which indicates the strand and the overall expectation to which the group of specific expectations corresponds (e.g., “B2” indicates that the group relates to overall expectation 2 in strand B). This organization is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups, nor is it intended to imply that learning the expectations happens in a linear, sequential way. The numbered headings are used merely as an organizational structure to help teachers focus on particular aspects of knowledge, concepts, and skills as they develop various lessons and learning activities for students. In the mathematics curriculum, additional subheadings are used within each group of expectations to identify the topics addressed in the strand.

The knowledge and skills described in the expectations in Strand A: Mathematical Thinking and Making Connections apply to all areas of course content and must be developed in conjunction with learning in strands B through F. Teachers should ensure that students develop the mathematics knowledge and skills in appropriate ways as they work to achieve the curriculum expectations in strands B through F. Students’ application of the knowledge and skills described in Strand A must be assessed and evaluated as part of their achievement of the overall expectations in strands B through F.

Note: Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics is an exception. It has a single overall expectation that is to be included in classroom instruction throughout the course, *but not in assessment, evaluation, or reporting*.

Teacher Supports

The expectations are accompanied by “teacher supports”, which may include examples, key concepts, teacher prompts, instructional tips, and/or sample tasks. These elements are intended to promote understanding of the intent of the specific expectations and are offered as illustrations for teachers. *The teacher supports do not set out requirements for student learning; they are optional, not mandatory*.

“Examples” are meant to illustrate the intent of the expectation, the kind of knowledge, concepts, or skills, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails.

“Key concepts” identify the central principles and mathematical ideas that underpin the learning in that specific expectation.

“Teacher prompts” are sample guiding questions and considerations that can lead to discussions and promote deeper understanding.

“Instructional tips” are intended to support educators in delivering instruction that facilitates student learning related to the knowledge, concepts, and skills set out in the expectations.

“Sample tasks” are developed to model appropriate practice for the course. They provide possible learning activities for teachers to use with students and illustrate connections between the mathematical knowledge, concepts, and skills. Teachers can choose to draw on the sample tasks that are appropriate for their classrooms, or they may develop their own approaches that reflect a similar level of complexity and high-quality mathematical instruction. Whatever the specific ways in which the requirements outlined in the expectations are implemented in the classroom, they must, wherever possible, be inclusive and reflect the diversity of the student population and the population of the province. When designing inclusive learning tasks, teachers reflect on their own biases and incorporate their deep knowledge of the curriculum, as well as their understanding of the diverse backgrounds, lived experiences, and identities of students. Teachers will notice that some of the sample tasks address the requirements of the expectation they are associated with and incorporate mathematical knowledge, concepts, or skills described in expectations in other strands of the course. Some tasks are cross-curricular in nature and will cover expectations in other disciplines in conjunction with the mathematics expectations.

The Mathematical Processes

Students learn and apply the mathematical processes as they work to achieve the expectations outlined in the curriculum. All students are actively engaged in applying these processes throughout the course.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

The mathematical processes can be understood as the processes through which all students acquire and apply mathematical knowledge, concepts, and skills. These processes are interconnected. Problem solving and communicating have strong links to all of the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to pose questions, make conjectures, and justify solutions, orally and in writing. The communication and reflection that occur before, during, and after the process of problem solving support students as they work to articulate and refine their thinking and to examine the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By understanding how others solve a problem, students can begin to reflect on their own thinking (a process known as “metacognition”) and the thinking of others, as well as their own language use (a process known as “metalinguistic awareness”), and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge, concepts, and skills that students acquire throughout the course. All students problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of mathematical concepts, and the skills required in all strands.

Problem Solving

Problem solving is central to doing mathematics. By learning to solve problems and by learning *through* problem solving, students are given, and create, numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs that place all students’ experiences and queries at the centre of mathematical learning. Therefore, problem solving should be the foundation of mathematical instruction. It is considered an essential process through which all students are able to achieve the expectations in mathematics and is an integral part of the Ontario mathematics curriculum.

Problem solving:

- increases opportunities for the use of critical thinking skills (e.g., selecting appropriate tools and strategies, estimating, evaluating, classifying, assuming, recognizing relationships, conjecturing, posing questions, offering opinions with reasons, making judgements) to develop mathematical reasoning;
- supports all students in developing their own mathematical identity;
- allows all students to use the varied mathematical knowledge and experiences they bring to school;
- supports all students in making connections among mathematical knowledge, concepts, and skills, and between situations inside and outside the classroom;
- has the potential to promote the collaborative sharing of ideas and strategies, and promotes talking about and interacting with mathematics;
- empowers students to use mathematics to address issues relevant to their lived realities;
- facilitates the use of creative-thinking skills when developing solutions and approaches;
- supports students in finding enjoyment in mathematics and becoming more confident in their ability to do mathematics.

Most importantly, when problem solving is done in a mathematical context relevant to students' experiences and/or derived from their own problem posing, it furthers their understanding of mathematics and develops their mathematical agency.

Problem-Solving Strategies. Problem-solving strategies are methods that can be used to solve problems of various types. Common problem-solving strategies include the following: simulating; making a model, picture, or diagram; using concrete materials; looking for a pattern; guessing and checking; making an organized list; making a table or chart; solving a simpler version of the problem; working backwards; and using logical reasoning. Teachers can support all students as they develop their use of these strategies by engaging with solving various kinds of problems – instructional problems, routine problems, and non-routine problems. As students develop their repertoire over time, they become more confident in posing their own questions, more mature in their problem-solving skills, and more flexible in using appropriate strategies when faced with new problem-solving situations.

Reasoning and Proving

Reasoning and proving are integral to mathematics and involve students using their understanding of mathematical knowledge, concepts, and skills to justify their thinking. Proportional reasoning, algebraic reasoning, spatial reasoning, statistical reasoning, and probabilistic reasoning are all forms of mathematical reasoning. Students also use their understanding of numbers and operations, geometric properties, and measurement relationships to reason through solutions to problems. Students develop algebraic reasoning by generalizing understanding of numbers and operations, properties, and relationships between quantities. They develop functional thinking by generalizing patterns and non-numeric sequences and using inverse operations. Students may need to identify assumptions in order to begin working on a solution. Teachers can provide all students with learning opportunities where they must form mathematical conjectures and then test or prove them to verify whether they hold true. Initially, students may rely on the viewpoints of others to justify a choice or an approach to a solution. As they develop their own reasoning skills, they will begin to justify or prove their solutions by providing evidence.

Reflecting

Students reflect when they are working through a problem to monitor their thought process, to identify what is working and what is not working, and to consider whether their approach is appropriate or whether there may be a more effective approach. Students also reflect after they have solved a problem by considering the reasonableness of their answer and whether adjustments need to be made. Teachers can support all students as they develop their reflecting and metacognitive skills by asking questions that have them examine their thought processes. In an inclusive learning environment, students also reflect on their peers' thinking processes to further develop deep understanding. Students can also reflect on how their new knowledge can be applied to past and future problems in mathematics.

Connecting

Experiences that allow all students to make connections – to understand, for example, how knowledge, concepts, and skills from one strand of mathematics are related to those from another – will support students in grasping general mathematical principles. Through making connections, students learn that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another, and to understand other disciplines. Recognizing the relationships between representations, concepts, and procedures also supports the development of deeper mathematical understanding. In addition, making connections between the mathematics they learn at school and its significance in their everyday lives supports students in deepening their understanding of mathematics and allows them to understand how useful and relevant it is in the world beyond the classroom.

Communicating

Communication is an essential process in learning mathematics. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, the whole class, a community member or group, or their family. They may use oral, visual, written, or gestural communication. Students also acquire the language of mathematics and develop their communication skills, which includes expressing, understanding, and using appropriate mathematical terminology, symbols, conventions, and models, through meaningful interactions with each other.

For example, teachers can ask students to:

- illustrate their mathematical understanding in various ways, such as with diagrams and representations;
- share and clarify their ideas, understandings, and solutions;
- create and defend mathematical arguments;
- provide meaningful descriptive feedback to peers;
- pose and ask relevant questions.

Communication also involves active listening and responding mindfully with an awareness of socio-cultural contexts. Using Culturally Responsive and Relevant Pedagogy, teachers provide opportunities for all students to contribute to discussions about mathematics in the classroom. Effective classroom communication requires a supportive and inclusive environment in which all members of the class are invited to participate and are valued when they speak and when they question, react to, and elaborate on the statements of their peers and the teacher.

Representing

Students represent mathematical ideas and relationships and model situations using tools, pictures, diagrams, graphs, tables, numbers, words, and symbols. Some students may also be able to use other languages and/or digital and multimodal resources. Teachers recognize and value the variety of representations that students use, as each student may have different prior access to and experiences with mathematics. While encouraging student engagement and affirming the validity of their representations, teachers support students in reflecting on the appropriateness of their representations and refining them. Teachers support students as they make connections among various representations that are relevant to both the student and the audience they are communicating with, so that all students can develop a deeper understanding of mathematical concepts and relationships. All students are supported in using the different representations appropriately and as needed to model situations, solve problems, and communicate their thinking.

Selecting Tools and Strategies

Students develop the ability to select appropriate tools, technology, and strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Tools. All students should be encouraged to select and use tools to illustrate mathematical ideas. Students come to understand that making their own representations is a powerful means of building understanding and of explaining their thinking to others. Using tools supports students as they:

- identify patterns and relationships;
- make connections between mathematical concepts and between concrete and abstract representations;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others, including by gesturing.

Technology. A wide range of technological and digital tools can be used in many contexts for students to interact with as they learn and extend concepts, and do mathematics.

Students can use:

- computers, calculators, probes, and computer algebra systems to perform complex operations; create graphs; and collect, organize, and display data;
- digital tools, apps, and social media to investigate mathematical concepts and develop an understanding of mathematical relationships;
- statistical software to manipulate, analyse, represent, sort, and communicate real-life data sets of all sizes;
- coding software to better understand the structures and relationships of mathematics;
- dynamic geometry software and online geometry tools to develop spatial sense;
- computer programs to represent and simulate mathematical situations (i.e., mathematical modelling);
- communications technologies to support and communicate their thinking and learning;

- computers, tablets, and mobile devices to access mathematical information available on the websites of organizations around the world in the language of instruction and/or other languages and to develop information literacy.

Developing the ability to perform mental computations is an important aspect of student learning in mathematics. Students must, therefore, use technology with discretion, when it makes sense to do so. When students use technology in their mathematics learning, they should apply mental computation, reasoning, and estimation skills to predict and check the reasonableness of answers.

Strategies. Problem solving often requires students to select an appropriate strategy. Students learn to use more efficient ways to reach a conclusion. For example, students can solve problems involving a linear relationship by extending a pattern using pictures, creating a table of values, or developing a general case and solving an equation. The selection of an appropriate strategy may also be based on feasibility. For example, students may choose to collect their own samples of data or access data collected in large amounts via computer programs.

Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics

Building social-emotional learning (SEL) skills in a secondary classroom involves continuing the development of students' self-awareness, self-management, social awareness, relationship skills, and responsible decision-making.[2] In this course, the focus is on the mathematical context and giving students the tools they need for success in their future mathematical learning, as they learn the skills for:

- recognizing and identifying emotions that support mathematical learning;
- recognizing sources of stress that present challenges to mathematical learning;
- identifying resources and supports that aid perseverance in mathematical learning;
- building healthy relationships and communicating effectively in mathematics;
- developing a healthy mathematical identity through building self-awareness;
- developing critical and creative mathematical thinking.

In an anti-racist and anti-discriminatory learning environment, explicit instruction, practice, modelling, self-reflection, and reinforcement both inside and outside the classroom make a difference in development of these skills. As with all instruction, continual consideration must be given to how educational systems and institutions can communicate and understand more inclusive perspectives on experiencing and displaying emotions, respect, and professionalism. SEL skills cannot be taught without

---

[2] CASEL (Collaborative for Academic, Social, and Emotional Learning), *Evidence-Based Social and Emotional Learning Programs: CASEL Criteria Updates and Rationale* (Chicago, IL: Author, 2020).

the context of systemic oppression and racism that many Ontario students navigate daily. Research has shown that educator bias can negatively affect the evaluation of social-emotional learning skills in relation to particular groups of students; for example, Indigenous students; Black and other racialized students; students with special education needs and disabilities; and students otherwise marginalized.

At the same time, there is strong evidence that teaching transformational social-emotional learning skills at school, when implemented in an anti-racist and anti-discriminatory, culturally responsive and relevant way, can contribute to students' overall health and well-being and to successful academic performance. Developing social-emotional learning skills also supports positive mental health, as well as students' ability to learn and experience academic success. Learning related to the overall expectation in this strand occurs in the context of learning related to the other six strands, and the focus is on intentional instruction only, not on assessment, evaluation, or reporting.

In order for SEL to be effective, teaching and learning approaches must consider and address the lived realities of students, including the ways in which educator biases affect students' experiences in the classroom. Approaches to support SEL instruction must be mediated through authentic and respectful conversations about students' lived realities. These realities may include the inequities students negotiate inside and outside the classroom, educator biases that perpetuate systemic racism, historical and intergenerational trauma related to the education system, institutional and interpersonal discrimination, and harassment.

Human rights principles[3] and the Education Act identify the importance of creating a climate of understanding of, and mutual respect for, the dignity and worth of each person, so that each person can contribute fully to the development and well-being of their community. Human rights law guarantees a person's right to equal treatment in education. It requires educators and school leaders to actively prevent all discrimination and harassment and respond appropriately when they do occur, to create an inclusive environment, to remove barriers that limit the ability of students, and to provide accommodations where necessary.

Intentional Instruction

Social-emotional learning skills can be developed across all subjects of the curriculum – including mathematics – as well as during various school activities, at home, and in the community. These skills support students in understanding and applying mathematical thinking and making connections across the course that are key to learning and doing mathematics. They support all students – and indeed all learners, including educators and parents – as they develop confidence, cope with challenges, and think critically. This in turn enables students to improve and demonstrate mathematics knowledge, concepts,

---

[3] More information on human rights in Ontario education is available in "Human Rights, Equity, and Inclusive Education" in the main "Considerations for Program Planning" section.

and skills in a variety of situations. Social-emotional learning skills support every student in developing a healthy identity as a capable mathematics learner.

Educator self-reflection on their own socio-cultural awareness is an essential component in the instruction of SEL in Ontario schools. Self-reflection is an important part of understanding oneself, one's identity and worldview, one's own beliefs, one's unconscious biases, one's privilege, and one's responses to these. For educators, self-awareness and self-reflection help to interrogate and understand their own position, as well as provide some grounding principles that can be used to support all students in enhancing their social-emotional learning skills while teaching in a way that is culturally responsive. Ensuring a culturally responsive and reflective approach that supports students in developing social-emotional learning skills begins with educator reflection and consideration of the learning environment. Educators reflect on instructional strategies, classroom climate, and the cultural context in which they teach, and consider making adjustments in any of these areas to more effectively support student learning and well-being for all students. SEL skills are developed within a learning context and with consideration of the individual student, and of their relationships to the classroom teacher, peers, other educators, the larger school community, and the world beyond.

Working with students to identify their personal learning goals related to SEL skills ensures that the intended learning is clear and transparent to all students and that all lived experiences are recognized. For example, when teachers are explicitly teaching skills for healthy relationships during problem-solving in mathematics, students and teachers work together to identify what these skills can look like and sound like. This may include recognizing different approaches to problem-solving that may be used in the students' homes or communities and in a variety of cultures; using encouraging words when communicating; and listening to each other about using different problem-solving approaches if the first one doesn't succeed. Educators model and teach these skills during instruction. Students may show their understanding of these skills in a variety of ways and reflect on their own progress individually.

Social-Emotional Learning Skills: Key Components and Sample Strategies

The chart below provides information about social-emotional learning skills, including key components and sample strategies in the context of mathematics learning.

Skills *What are the skills? How do they help? What do they look like in mathematics?*	**Key Components and Sample Strategies**
Recognizing and Identifying Emotions That Support Mathematical Learning Students often experience a range of emotions over the course of their day at school. They may feel happy, sad, angry, frustrated, or excited, or any number of emotions in combination. Students may struggle to identify and appropriately express their feelings. Learning to recognize	• Recognizing a range of emotions in self and others • Understanding connections between thoughts, feelings, and actions and the impacts of each of these on the others • Recognizing that new or challenging learning may involve a sense of

different emotions can support students in interacting with mathematical content and within mathematical learning communities in healthy ways. When students understand the influence of thoughts and emotions on behaviour, they can improve the quality of their interactions and are better able to respond to themselves and others in ways that are compassionate and caring, and that honour their own social and emotional needs. In mathematics, as they learn new mathematics concepts and interact with others while problem solving, students have many opportunities to develop awareness of their emotions and to use communication skills to express their feelings and to respond with care when they recognize emotions in others.	excitement or an initial sense of discomfort • Applying strategies such as: ○ identifying, naming, and reasoning through the cause of particular emotions ○ using language such as "I'm feeling frustrated because…" ○ using tools (e.g., pictures) and language to gauge intensity of emotion
Recognizing Sources of Stress That Present Challenges to Mathematical Learning Every day, students are exposed to a range of challenges that can contribute to feelings of stress. As they learn stress management and coping skills, they come to recognize that stress is a part of learning and life and that it can be managed. While taking steps to dismantle systemic barriers to student well-being and success, educators can support students as they learn ways to respond to challenges in mathematics learning that enable them to "bounce back" and, in this way, build resilience in the face of life's obstacles. Over time, with support, practice, feedback, reflection, and experience, students begin to build a set of personal coping strategies that they can carry with them through life. In mathematics, students work through challenging problems, understanding that their resourcefulness in using coping strategies strengthens their personal resilience.	• Seeking support from peers, teachers, family, or their extended community • Applying strategies such as: ○ "chunking" a task or problem into manageable components and tackling one piece at a time ○ thinking of a similar problem ○ engaging in guided imagery and visualization ○ stretching ○ pausing and reflecting ○ using an iterative approach to solve a problem, including reframing questions, trying out different methods, estimating, and guessing and checking solutions

Identifying Resources and Supports That Aid Perseverance in Mathematical Learning
 	
In a supportive and inclusive environment, students have regular opportunities to practise and apply perseverance skills as they solve mathematical problems and develop an appreciation for learning from mistakes as a part of the mathematics learning process. Educators can support students in approaching challenges in life with an understanding that there is struggle in most successes and that accessing the right support can lead to success. To that end, students need to identify and access educators as key resources. Through regular interactions, students and educators can build relationships based on trust and respect. Educators can also support students in noticing and naming harmful classroom interactions such as microaggressions and discrimination, and can support them when they report incidents of harm. While building skills for perseverance can have an impact on an individual student level, it is important to recognize the critical role educators play when they actively take steps to acknowledge and address systemic barriers at all levels (in the classroom, in the school, across the system, in the community) that hinder mathematical learning for students.
	• Embracing mistakes as a necessary and helpful part of learning
	• Noticing strengths and positive aspects of experiences, appreciating the value of practice
	• Creating a list of supports and resources, including people, that can aid them in persevering
	• Applying strategies such as:
	  ○ supporting peers by encouraging them to keep trying if they make a mistake
	  ○ using personal affirmations like "I can do this."

Building Healthy Relationships and Communicating Effectively in Mathematics
 	
Healthy relationships are at the core of developing and maintaining physically and mentally safe, healthy, equitable, caring school and classroom communities. When students interact in meaningful ways with others, mutually respecting diversity of thought and expression, their sense of belonging within the school and community is enhanced. Learning healthy relationship skills helps students establish patterns of effective communication and inspires healthy, cooperative relationships. These skills include the ability to understand and appreciate another person's perspective, to empathize with others, to listen attentively, and to resolve conflict in healthy ways. In mathematics, students have opportunities to develop and practise skills that support healthy interaction with others as they work together in small groups or in pairs to solve math problems and confront challenges. Developing
	• Recognizing and understanding the impact of one's emotions and actions on others
	• Listening attentively
	• Considering other ideas and perspectives
	• Practicing empathy and care
	• Using conflict-resolution skills
	• Using cooperation and collaboration skills
	• Applying strategies such as:
	  ○ seeking opportunities to help others
	  ○ working as part of a team and playing different roles (e.g., leader, scribe or illustrator, data collector, observer) that contribute to outcomes in different ways

these skills helps students to communicate with teachers, peers, and family about mathematics with an appreciation of the beauty and wonder of mathematics.	

Developing a Healthy Mathematical Identity Through Building Self-Awareness
 	
Knowing who we are and having a sense of purpose and meaning in our lives enables us to function in the world as self-aware individuals. Our sense of identity enables us to make choices that support our well-being and allows us to connect with and have a sense of belonging in various cultural and social communities. Educators should note that for First Nations, Métis, and Inuit students, the term “sense of identity and belonging” may also mean belonging to and identifying with a particular community and/or nation. Self-awareness and identity skills supports students in exploring who they are – their strengths, preferences, interests, values, and ambitions – and how their social and cultural contexts have influenced them. This exploration is grounded in affirming cultural heritage, considering social identities, and assessing the impact of beliefs and biases. In mathematics, as they learn new skills, students use self-awareness skills to monitor their progress and identify their individual strengths and gifts; in the process, they build their identity as mathematics learners who are capable of actualizing their individual pathways.
	• Knowing oneself
	• Caring for oneself
	• Having a sense of mattering and of purpose
	• Identifying personal strengths
	• Having a sense of belonging and community
	• Communicating their thinking and feelings about mathematics
	• Applying strategies such as:
	    ○ building their identity as a math learner as they learn independently as a result of their efforts and challenges
	    ○ monitoring progress in skill development
	    ○ reflecting on strengths and accomplishments and sharing these with peers or caring adults

Developing Critical and Creative Mathematical Thinking
 	
Critical and creative thinking skills enable us to make informed judgements and decisions on the basis of a clear and full understanding of ideas and situations, and their implications, in a variety of settings and contexts. Students learn to question, interpret, predict, analyse, synthesize, detect bias, and distinguish between alternatives. They practise making connections, setting goals, creating plans, making and evaluating decisions, and analysing and solving problems for which there may be no clearly defined answers. In all aspects of the mathematics curriculum, students have opportunities to develop critical and creative thinking skills. Students have opportunities to build on prior learning, go deeper, and make personal connections through real-life applications.
	• Making connections
	• Making decisions
	• Evaluating choices, reflecting on and assessing strategies
	• Communicating effectively
	• Managing time
	• Setting goals and making plans
	• Applying organizational skills
	• Applying strategies such as:
	    ○ determining what is known and what needs to be found
	    ○ using various webs, charts, diagrams, and representations to help identify connections and interrelationships
	    ○ using organizational strategies and tools, such as planners,

	trackers, and goal-setting frameworks

The Strands in the Grade 9 Mathematics Course

The Grade 9 mathematics course is designed to be inclusive of all students in order to facilitate their transition to learning at the secondary level by offering opportunities to broaden their knowledge and skills in mathematics. This approach allows students to make informed decisions in choosing future mathematics courses based on their interests and on requirements for future jobs, trades, and professions.

The Grade 9 mathematics course is organized into seven strands. Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics focuses on a set of skills to be developed in the context of learning across all other strands. Strand A focuses on developing mathematical thinking and making connections to students' lived experiences as well as connecting curriculum to real-life applications as students acquire the mathematical concepts and skills set out in strands B through F. The remaining strands cover the interrelated content areas of number, algebra, data, geometry and measurement, and financial literacy. The Grade 9 mathematics course consolidates learning from the elementary grades and sets a foundation for learning in future secondary mathematics courses. The strands of the elementary mathematics program are closely aligned with those of the Grade 9 mathematics course, as shown in the following chart.

Elementary Mathematics	Grade 9 Mathematics
	AA. Social-Emotional Learning (SEL) Skills in Mathematics
A. Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes	A. Mathematical Thinking and Making Connections
B. Number	B. Number
C. Algebra	C. Algebra
D. Data	D. Data
E. Spatial Sense	E. Geometry and Measurement
F. Financial Literacy	F. Financial Literacy

Strands in Grade 9 Mathematics
Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics
Strand A: Mathematical Thinking and Making Connections • Mathematical processes • Making connections
Strand B: Number • Development and use of numbers • Number sets • Powers • Rational numbers • Applications
Strand C: Algebra • Development and use of algebra • Algebraic expressions and equations • Coding • Application of linear and non-linear relations • Characteristics of linear and non-linear relations
Strand D: Data • Application of data • Representation and analysis of data • Application of mathematical modelling • Process of mathematical modelling
Strand E: Geometry and Measurement • Geometric and measurement relationships
Strand F: Financial Literacy • Financial decisions

Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics

This strand comprises a single overall expectation that is to be included in classroom instruction throughout the course, but not in assessment, evaluation, or reporting. Students are supported in exploring social-emotional learning skills in mathematics.

Strand A: Mathematical Thinking and Making Connections

Throughout the course, students apply the mathematical processes to develop conceptual understanding and procedural fluency while they engage in learning related to strands B through F. They also make connections between the mathematics they are learning and their lived experiences, various knowledge systems, and real-life applications, including employment and careers.

Strand B: Number

In this strand, students continue to make connections among various number systems, the cultural development of number concepts, and real-life applications. They will extend their learning about positive fractions, positive decimal numbers, and integers to work with negative fractions and negative decimal numbers. Students also extend their knowledge and skills related to percentages, ratios, rates, and proportions to make further connections to real life.

Strand C: Algebra

In this strand, students continue to develop an understanding of algebra by making connections between algebra and numbers as they generalize relationships with algebraic expressions and equations. Students will extend and apply coding skills to dynamically represent situations, analyse mathematics concepts, and solve problems in various contexts. Students will be introduced to various representations of linear and non-linear relations that they will study in more depth in future secondary mathematics courses. Students develop an understanding of constant rate of change and initial values of linear relations, and solve related real-life problems.

Strand D: Data

In this strand, students extend their data literacy skills to examine the collection, representation, and use of data, as well as their implications in various contexts. Students consolidate and extend their understanding of data involving one and two variables and its connections to real life. Using data, students continue to apply the process of mathematical modelling to analyse real-life situations.

Strand E: Geometry and Measurement

In this strand, students make connections among various geometric properties and their real-life applications. Students analyse and create designs to extend their understanding of geometric relationships to include circle and triangle properties. Students solve problems using different units within and between various measurement systems, examine the relationships between the volume of cones and cylinders and of pyramids and prisms, and solve problems that involve the application of perimeter, area, surface area, and volume.

Strand F: Financial Literacy

In this strand, students analyse financial situations to explain how mathematics can be used to understand such situations and inform financial decisions. They extend their financial literacy knowledge to answer questions related to appreciation and depreciation, and explain how budgets can be modified based on changes in circumstances. Students compare the effects of different interest rates, down payments, and other factors associated with purchasing goods and services. Students use their learning from other strands to solve financial problems of interest.

Some Considerations for Program Planning

Teachers consider many factors when planning a mathematics program that cultivates an inclusive environment in which all students can maximize their mathematical learning. This section highlights the key strategies and approaches that teachers and school leaders should consider as they plan effective and inclusive mathematics programs. Additional information can be found in the "Considerations for Program Planning" section, which provides information applicable to all curricula.

Instructional Approaches in Mathematics

Instruction in mathematics should support all students in acquiring the knowledge, skills, and habits of mind that they need in order to achieve the curriculum expectations and be able to enjoy and participate in mathematics learning for years to come.

Effective mathematics instruction begins with knowing the complex identities and profiles of the students, having high academic expectations for and of all students, providing supports when needed, and believing that all students can learn and do mathematics. Teachers incorporate Culturally Responsive and Relevant Pedagogy (CRRP) and provide authentic learning experiences to meet individual students' learning strengths and needs. Effective mathematics instruction focuses on the development of conceptual understanding and procedural fluency, skill development, and communication, as well as problem-solving skills. It takes place in a safe and inclusive learning environment, where all students are valued, empowered, engaged, and able to take risks, learn from mistakes, and approach the learning of mathematics in a confident manner. Instruction that is student centred and asset based builds effectively on students' strengths to develop mathematical habits of mind, such as curiosity and open-mindedness; a willingness to question, to challenge and be challenged; and an awareness of the value of listening intently, reading thoughtfully, and communicating with clarity.

Learning should be relevant: embedded in the lived realities of all students and inspired by authentic, real-life contexts as much as possible. This approach allows students to develop key mathematical concepts and skills, to appreciate the beauty and wide-ranging nature of mathematics, and to realize the potential of mathematics to raise awareness and effect social change that is innovative and sustainable. A focus on making learning relevant supports students in their use of mathematical reasoning to make connections throughout their lives.

Universal Design for Learning (UDL) and Differentiated Instruction (DI)

Students in every mathematics classroom vary in their identities, lived experiences, personal interests, learning profiles, and readiness to learn new concepts and skills. Universal Design for Learning (UDL) and differentiated instruction (DI) are robust and powerful approaches to designing assessment and instruction to engage all students in mathematical tasks that develop conceptual understanding and procedural fluency. Providing each student with opportunities to be challenged and to succeed requires teachers to attend to student differences and provide flexible and responsive approaches to instruction. UDL and DI can be used in combination to help teachers respond effectively to the strengths and needs of all students.

The aim of the UDL framework is to assist teachers in designing mathematics programs and environments that provide all students with equitable access to the mathematics curriculum. Within this framework, teachers engage students in multiple ways in order to support them in becoming purposeful and motivated in their mathematics learning. Teachers take into account students' diverse learner profiles by designing tasks that offer individual choice, ensuring relevance and authenticity, providing graduated levels of challenge, and fostering collaboration in the mathematics classroom. Teachers also represent concepts and information in multiple ways to help students become resourceful and knowledgeable learners. For example, teachers use a variety of media to ensure that students are provided with alternatives for auditory and visual information; they clarify mathematics vocabulary and symbols; and they highlight patterns and big ideas to guide information processing. To support learners as they focus strategically on their learning goals, teachers create an environment in which learners can express themselves using a range of kinesthetic, visual, and auditory strengths. For example, teachers can improve access to tools or assistive devices; vary ways in which students can respond and demonstrate their understanding of concepts; and support students in goal-setting, planning, and time-management skills related to their mathematics learning.

Designing mathematics tasks through UDL allows the learning to be "low floor, high ceiling" – that is, all students are provided with the opportunity to find their own entry point to the learning. Teachers can then support students in working at their own pace and provide further support as needed, while continuing to move student learning forward. Tasks that are intentionally designed to be low floor, high ceiling provide opportunities for students to use varied approaches and to continue to be engaged in learning with varied levels of complexity and challenge. This is an inclusive approach that is grounded in a growth mindset: the belief that everyone can do well in mathematics.

While UDL provides teachers with broad principles for planning mathematics instruction and learning experiences for a diverse group of students, DI allows them to address specific skills and learning needs. DI is rooted in assessment and involves purposefully planning varied approaches to teaching the content of the curriculum; to the processes (e.g., tasks and activities) that support students as they make sense of what they are learning; to the ways in which students demonstrate their learning and the outcomes they are expected to produce; and to the learning environment. DI is student centred and involves a strategic blend of whole-class, small-group, and individual learning activities to suit students' differing strengths, interests, and levels of readiness to learn. Attending to students' varied readiness for learning mathematics is an important aspect of differentiated teaching. Learners who are ready for greater challenges need support in aiming higher, developing belief in excellence, and co-creating problem-based tasks to increase the complexity while still maintaining joy in learning. Students who are struggling to learn a concept need to be provided with the scaffolding and encouragement to reach high standards. Through an asset-based approach, teachers focus on these learners' strengths, imbuing instructional approaches with a strong conviction that all students can learn. To make certain concepts more accessible, teachers can employ strategies such as offering students choice, and providing open-ended problems that are based on relevant real-life situations and supported with visual and hands-on learning. Research indicates that using differentiated instruction in mathematics classrooms can diminish inequities.

Universal Design for Learning and differentiated instruction are integral aspects of an inclusive mathematics program and the achievement of equity in mathematics education. More information on these approaches can be found in the ministry publication *Learning for All: A Guide to Effective Assessment and Instruction for All Students, Kindergarten to Grade 12* (2013).

High-Impact Practices

Teachers understand the importance of knowing the identities and profiles of all students and of choosing the instructional approaches that will best support student learning. The approaches that teachers employ vary according to both the learning outcomes and the needs of the students, and teachers choose from and use a variety of accessible, equitable high-impact instructional practices.

The thoughtful use of these high-impact instructional practices – including knowing when to use them and how they might be combined to best support the achievement of specific math goals – is an essential component of effective math instruction. Researchers have found that the following practices consistently have a high impact on teaching and learning mathematics:[4]

- **Learning Goals, Success Criteria, and Descriptive Feedback.** Learning goals and success criteria outline the intention for the lesson and how this intention will be achieved to ensure teachers and students have a clear and common understanding of what is being learned and what success looks like. The use of descriptive feedback involves providing students with the precise information they need in order to reach the intended learning goal.
- **Direct Instruction.** This is a concise, intentional form of instruction that begins with a clear learning goal. It is not a lecture or a show-and-tell. Instead, direct instruction is a carefully planned and focused approach that uses questioning, activities, or brief demonstrations to guide learning, check for understanding, and make concepts clear. Direct instruction prioritizes

[4] John Hattie, Douglas Fisher, Nancy Frey, Linda M. Gojak, Sara Delano Moore, and William Mellman, *Visible Learning for Mathematics: What Works Best to Optimize Student Learning, Grades K–12* (Thousand Oaks, CA: Corwin Mathematics, 2017).

feedback and formative assessment throughout the learning process and concludes with a clear summary of the learning that can be provided in written form, orally, and/or visually.

- **Problem-Solving Tasks and Experiences.** It is an effective practice to use a problem, intentionally selected or created by the teacher or students, to introduce, clarify, or apply a concept or skill. This practice provides opportunities for students to demonstrate their agency by representing, connecting, and justifying their thinking. Students communicate and reason with one another and generate ideas that the teacher connects in order to highlight important concepts, refine existing understanding, eliminate unsuitable strategies, and advance learning.
- **Teaching about Problem Solving.** Teaching students about the process of problem solving makes explicit the critical thinking that problem solving requires. It involves teaching students to identify what is known and unknown, to draw on similarities and differences between various types of problems, and to use representations to model the problem-solving situation.
- **Tools and Representations.** The use of a variety of appropriate tools and representations supports a conceptual understanding of mathematics. Carefully chosen and used effectively, representations and tools such as manipulatives make math concepts accessible to a wide range of learners. At the same time, student interactions with representations and tools also give teachers insight into students' thinking and learning.
- **Math Conversations.** Effective mathematical conversations create opportunities for all students to express their mathematical thoughts and to engage meaningfully in mathematical talk by listening to and responding to the ideas of others. These conversations involve reasoning, proving, building on the thinking of others, defending and justifying their own thinking, and adjusting their perspectives as they build their mathematical understanding, confidence, and awareness of the mathematical thoughts of others.
- **Small-Group Instruction.** A powerful strategy for moving student learning forward, small-group instruction involves targeted, timely, and scaffolded mathematics instruction that meets the learning needs of specific students at appropriate times. By working with small and flexible groups, whether they are homogenous or heterogenous, teachers can personalize learning in order to close gaps that exist or extend thinking. Small-group instruction also provides opportunities for teachers to connect with and learn more about student identities, experiences, and communities, which the teachers can build on as a basis for their mathematics instruction.
- **Deliberate Practice.** Practice is best when it is purposeful and spaced over time. It must always follow understanding and should be continual and consistent. Teachers provide students with timely descriptive feedback to ensure that students know they are practising correctly and sufficiently. Students also need to practise metacognition, or reflecting on their learning, in order to become self-directed learners.
- **Flexible Groupings.** The intentional combination of large-group, small-group, partnered, and independent working arrangements, in response to student and class learning needs, can foster a rich mathematical learning environment. Creating flexible groupings in a mathematics class enables students to work independently of the teacher but with the support of their peers, and it strengthens collaboration and communication skills. Regardless of the size of the group, it is of utmost importance that individual students have ownership of their learning.

While a lesson may prominently feature one of these high-impact practices, other practices will inevitably also be involved. The practices are rarely used in isolation, nor is there any single "best" instructional practice. Teachers strategically choose the right practice, for the right time, in order to create an optimal learning experience for all students. They use their socio-cultural awareness of themselves and their students, a deep understanding of the curriculum and of the mathematics that underpins the expectations, and a variety of assessment strategies to determine which high-impact instructional practice, or combination of practices, best supports the students. These decisions are made continually throughout a lesson. The appropriate use of high-impact practices plays an important role in supporting student learning.

More information can be found in the resource section on high-impact practices in mathematics.

When teachers effectively implement Universal Design for Learning, differentiated instruction, and high-impact practices in mathematics programs, they create opportunities for students to develop mathematics knowledge and skills, to apply mathematical processes, and to develop transferable skills that can be applied in other curricular areas.

The Role of Information and Communication Technology in Mathematics

The mathematics curriculum was developed with the understanding that the strategic use of technology is part of a balanced mathematics program. Technology can extend and enrich teachers' instructional strategies to support all students' learning in mathematics. Technology, when used in a thoughtful manner, can support and foster the development of mathematical reasoning, problem solving, and communication. For some students, technology is essential and required to access curriculum.

When using technology to support the teaching and learning of mathematics, teachers consider the issues of student safety, privacy, ethical responsibility, equity and inclusion, and well-being.

The strategic use of technology to support the achievement of the curriculum expectations requires a strong understanding of:

- the mathematical concepts being addressed;
- high-impact teaching practices that can be used, as appropriate, to achieve the learning goals;
- the capacity of the chosen technology to augment the learning, and how to use this technology effectively.

Technology (e.g., digital tools, computation devices, calculators, data-collection programs and coding environments) can be used specifically to support students' thinking in mathematics, to develop conceptual understanding (e.g., visualization using virtual graphing or geometry tools), and to facilitate access to information and allow better communication and collaboration (e.g., collaborative documents and web-based content that enable students to connect with experts and other students; language translation applications).

Coding has been introduced into the Grade 9 mathematics course as a continuum from the elementary mathematics curriculum. The elementary mathematics curriculum outlines a developmental progression for students to develop foundational coding skills. In Grade 9, students transition to using coding as a tool to interact with the mathematics they are learning. They use the skills developed in elementary to create and alter code in a multitude of coding environments including text-based programming languages, spreadsheets, computer algebra systems (CAS), and virtual graphing and geometry tools.

Technology can support English language learners in accessing mathematics terminology and ways of solving problems in their first language. Assistive technologies are critical in enabling some students with special education needs to have equitable access to the curriculum and in supporting their learning, and must be provided in accordance with a student's Individual Education Plan (IEP).

Technologies are important problem-solving tools. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the learning tools that may be helpful or necessary for them as they search for their own solutions to problems.

Teachers understand the importance of technology and how it can be used to access and support learning for all students. Additional information can be found in the "The Role of Information and Communications Technology" subsection of "Considerations for Program Planning".

Education and Career/Life Planning

Education and career/life planning supports students in their transition from secondary school to their initial postsecondary destinations, whether in apprenticeships, college, community living, university, or the workplace.

Mathematics teachers can support students in education and career/life planning by making authentic connections between the mathematics concepts students are learning in school and the knowledge and skills needed in different careers. These connections engage students' interest and allow them to develop an understanding of the usefulness of mathematics in the daily lives of workers.

Teachers can promote students' awareness of careers involving mathematics by exploring real-life applications of mathematics concepts and providing opportunities for career-related project work. Such activities allow students to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Community members can also act as a valuable resource by sharing their career expertise and supporting students in understanding the relevance of mathematics to various fields of study and careers. Career fairs, guest speakers, and job-shadowing days can provide opportunities for students to identify and explore mathematics-related careers.

Students may need support to comprehend the wide variety of professions and careers where mathematical concepts and processes are used. For example:

- fractions and imperial measures are used in various trades and daily activities;
- rates and percentages are used in banking, investing, and currency exchange;
- ratios and proportions are used in architecture, engineering, construction, nursing, pharmacy practice, hair colouring techniques, and fields related to culinary arts;
- algebraic reasoning is used in the sciences and computer programming;
- geometry and measurement concepts are used in construction, civil engineering, and art; statistics are used in real estate, the retail sector, tourism and recreation, conservation, finance, insurance, sports management, and research.

Students should be made aware that mathematical literacy, problem solving, and the other skills and knowledge they learn in the mathematics classroom are valuable assets in an ever-widening range of jobs and careers in today's society. More information can be found in the "Education and Career/Life Planning" subsection of "Considerations for Program Planning".

Planning Mathematics Programs for Students with Special Education Needs

Classroom teachers hold high expectations of all students and are the key educators in designing and supporting mathematics assessment and instruction for students with special education needs. They have a responsibility to support all students in their learning and to work collaboratively with special education teachers, where appropriate, to plan, design and implement appropriate instructional and assessment accommodations and modifications in the mathematics program to achieve this goal. More information on planning for and assessing students with special education needs can be found in the "Planning for Students with Special Education Needs" subsection of "Considerations for Program Planning".

Principles for Supporting Students with Special Education Needs

The following principles[5] guide teachers in effectively planning and teaching mathematics programs to students with special education needs, and also benefit all students:

[5] Adapted from Ontario Ministry of Education, *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, Kindergarten to Grade 6* (Toronto, ON: Author, 2005).

- The teacher plays a critical role in student success in mathematics.
- It is important for teachers to develop an understanding of the general principles of how students learn mathematics.
- The learning expectations outline interconnected, developmentally appropriate key concepts and skills of mathematics across all of the strands.
- It is important to support students in making connections between procedural knowledge and conceptual understanding of mathematics.
- The use of concrete, visual, and virtual representations and tools is fundamental to learning mathematics and provides a way of representing both concepts and student understanding.
- The teaching and learning process involves ongoing assessment. Students with special education needs should be provided with various opportunities to demonstrate their learning and thinking in multiple ways.

An effective mathematics learning environment and program that addresses the mathematical learning needs of students with special education needs is purposefully planned with the principles of Universal Design for Learning in mind and integrates the following elements:

- knowing the student's cultural and linguistic background, strengths, interests, motivations, and needs in mathematics learning in order to differentiate learning and make accommodations and modifications as outlined in the student's Individual Education Plan;
- building the student's confidence and positive identity as a mathematics learner;
- valuing the student's prior knowledge and connecting what the student knows with what the student needs to learn;
- identifying and focusing on the connections between broad concepts in mathematics;
- connecting mathematics with familiar, relevant, everyday situations and providing rich and meaningful learning contexts;
- fostering a positive attitude towards mathematics and an appreciation of mathematics through multimodal means, including through the use of assistive technology and the performance of authentic tasks;
- implementing research-informed instructional approaches (e.g., Concrete – Semi-Concrete – Representational – Abstract) when introducing new concepts to promote conceptual understanding, procedural accuracy, and fluency;
- creating a balance of explicit instruction, problem solving within a student's zone of proximal development, learning in flexible groupings, and independent learning. Each instructional strategy should take place in a safe, supportive, and stimulating environment while taking into consideration that some students may require more systematic and intensive support, and more explicit and direct instruction, before engaging in independent learning;
- assessing student learning through observations, conversations with the students, and frequent use of low-stakes assessment check-ins and tools;
- providing immediate feedback in order to facilitate purposeful, correct practice that supports understanding of concepts and procedures, as well as efficient strategies;
- providing environmental, assessment, and instructional accommodations in order to maximize the student's learning (e.g., making available learning tools such as virtual manipulatives,

computer algebra systems, and calculators; ensuring access to assistive technology), as well as modifications that are specified in the student's Individual Education Plan;
- building an inclusive community of learners and encouraging students with special education needs to participate in various mathematics-oriented class projects and activities;
- building partnerships with administrators and other teachers, particularly special education teachers, where available, to share expertise and knowledge of the curriculum expectations; co-develop content in the Individual Education Plan that is specific to mathematics; and systematically implement intervention strategies, as required, while making meaningful connections between school and home to ensure that what the student is learning in the school is relevant and can be practised and reinforced beyond the classroom.

Planning Mathematics Programs for English Language Learners

English language learners are working to achieve the curriculum expectations in mathematics while they are developing English-language proficiency. An effective mathematics program that supports the success of English language learners is purposefully planned with the following considerations in mind.

- Students' various linguistic identities are viewed as a critical resource in mathematics instruction and learning. Recognizing students' language resources and expanding linguistic competence enables students to use their linguistic repertoire in a fluid and dynamic way, mixing and meshing languages to communicate. This translingual practice[6] is creative and strategic, and allows students to communicate, interact, and connect with peers and teachers using the full range of their linguistic repertoire, selecting features and modes that are most appropriate to communicate across a variety of purposes, such as when developing conceptual knowledge and when seeking clarity and understanding.
- Students may be negotiating between school-based mathematics and ways of mathematical reasoning from diverse cultural and linguistic backgrounds. They may have deep mathematical knowledge and skills developed in another educational cultural and/or linguistic context, and may already have learned the same mathematical terms and concepts that they are studying now, but in another language.
- Knowledge of the diversity among English language learners and of their mathematical strengths, interests, and identities, including their social and cultural backgrounds, is important. These

[6] Ofelia García, Susana Ibarra Johnson, and Kate Seltzer, *The Translanguaging Classroom: Leveraging Student Bilingualism for Learning* (Philadelphia, PA: Caslon, 2017).

Sunny Man Chu Lau and Saskia Van Viegen, *Plurilingual Pedagogies* (Springer Cham, New York City, NY: 2020).

"funds of knowledge"[7] are historically and culturally developed skills and assets that can be incorporated into mathematics learning to create a richer and more highly scaffolded learning experience for all students, promoting a positive, inclusive teaching and learning environment. Understanding how mathematical concepts are described in students' home languages and cultures can provide insight into how students are thinking about mathematical ideas.[8]

- In addition to assessing their level of English-language proficiency, an initial assessment of the mathematics knowledge and skills of newcomer English language learners is required in Ontario schools.
- Differentiated instruction is essential in supporting English language learners, who face the dual challenge of learning new conceptual knowledge while acquiring English-language proficiency. Designing mathematics learning to have the right balance for English language learners is achieved through program adaptations (e.g., accommodations that utilize their background knowledge in their first language) that ensure the tasks are mathematically challenging, reflective of learning demands within the mathematics curriculum, and comprehensible and accessible to English language learners. Using the full range of a student's language assets, including additional languages that a student speaks, reads, and writes, as a resource in the mathematics classroom supports access to their prior learning, reduces the language demands of the mathematics curriculum, and increases engagement.
- Working with students and their families and with available community supports allows for the multilingual representation of mathematics concepts to create relevant and real-life learning contexts and tasks.

In a supportive learning environment, scaffolding the learning of mathematics assessment and instruction offers English language learners the opportunity to:

- integrate their linguistic repertoire rather than engage in language separation, and select and use the linguistic features and modes that are most appropriate for their communication purposes;
- discuss how mathematical concepts are described in their language(s) and cultures;[9]

[7] Elizabeth Marshall and Kelleen Toohey, "Representing Family: Community Funds of Knowledge, Bilingualism, and Multimodality," *Harvard Educational Review* 80, no. 2 (2010), 221–42.

[8] Lisa Lunney Borden, "What's the Word for…? Is There a Word for…? How Understanding Mi'kmaw Language Can Help Support Mi'kmaw Learners in Mathematics", *Mathematics Education Research Journal* 25, no. 1 (2013): 5–22.

[9] Lisa Lunney Borden, "What's the Word for…? Is There a Word for…? How Understanding Mi'kmaw Language Can Help Support Mi'kmaw Learners in Mathematics", *Mathematics Education Research Journal* 25, no. 1 (2013): 5–22.

- draw on their additional language(s) (e.g., some newcomer students may use technology to access mathematical terminology and ways of solving problems in their first language), prior learning experiences, and background knowledge in mathematics;
- learn new mathematical concepts in authentic, meaningful, and familiar contexts;
- engage in open and parallel tasks to allow for multiple entry points for learning;
- work in a variety of settings that support co-learning and multiple opportunities for practice (e.g., with partners or in small groups with same-language peers, as part of cooperative or collaborative learning, in group conferences);
- access the language of instruction during oral, written, and multimodal instruction and assessment, during questioning, and when encountering texts, learning tasks, and other activities in mathematics;
- use oral language in different strategically planned activities, such as "think-pair-share", "turn-and-talk", and "adding on", to express their ideas and engage in mathematical discourse;
- develop both everyday and academic vocabulary, including specialized mathematics vocabulary in context, through rephrasing and recasting by the teacher and through using student-developed bilingual word banks or glossaries;
- practise using sentence frames adapted to their English-language proficiency levels to describe concepts, provide reasoning, hypothesize, make judgements, and explain their thinking;
- use a variety of concrete and/or digital learning tools to demonstrate their learning in mathematics in multiple ways (e.g., orally, visually, kinesthetically), through a range of representations (e.g., portfolios, displays, discussions, models), and in multiple languages (e.g., multilingual word walls and anchor charts);
- have their learning assessed in terms of the processes they use in multiple languages, both during the learning and through teachers' observations and conversations.

Strategies used to differentiate instruction and assessment for English language learners in the mathematics classroom also benefit many other learners in the classroom, since programming is focused on leveraging all students' strengths, meeting learners where they are in their learning, being aware of language demands in mathematics, and making learning visible. For example, different cultural approaches to solve mathematical problems can help students make connections to the Ontario curriculum and provide classmates with alternative ways of solving problems.

English language learners in English Literacy Development (ELD) programs require accelerated support to develop both their literacy skills and their numeracy skills. These students have significant gaps in their education because of limited or interrupted opportunities for or access to schooling. In order to build a solid foundation of mathematics, they are learning key mathematical concepts missed in prior years. At the same time, they are learning the academic language of mathematics in English while not having acquired developmentally appropriate literacy skills in their first language. Programming for these students is therefore highly differentiated and intensive. These students often require focused support over a longer period than students in English as a Second Language (ESL) programs. The use of students' oral competence in languages other than English is a non-negotiable scaffold. The strategies described above, such as the use of visuals, the development of everyday and academic vocabulary, the use of technology, and the use of oral competence, are essential in supporting student success in ELD programs and in mathematics.

Supporting English language learners is a shared responsibility. Collaboration with administrators and other teachers, particularly ESL/ELD teachers, and Indigenous representatives, where possible, contributes to creating equitable outcomes for English language learners. Additional information on planning for and assessing English language learners can be found in the “Planning for English Language Learners” subsection of “Considerations for Program Planning”.

Cross-Curricular and Integrated Learning in Mathematics

When planning an integrated mathematics program, educators should consider that, although the mathematical content in the curriculum is outlined in discrete strands, students develop mathematical thinking, such as proportional reasoning, algebraic reasoning, and spatial reasoning, that transcends the expectations in the strands and even connects to learning in other subject areas. By purposefully drawing connections across all areas of mathematics and other subject areas, and by applying learning to relevant real-life contexts, teachers extend and enhance student learning experiences and deepen their knowledge and skills across disciplines and beyond the classroom.

For example, proportional reasoning, which is developed through the study of ratios and rates in the Number strand, is also used when students are working towards meeting learning expectations in other strands of the math curriculum, such as in Geometry and Measurement and in Algebra, and in other disciplines, such as science, geography, and the arts. Students then apply this learning in their everyday lives – for example, when adjusting a recipe, preparing a mixture or solutions, or making unit conversions.

Similarly, algebraic reasoning is applied beyond the Number and Algebra strands. For example, it is applied in measurement when learning about formulas, such as $Volume\ of\ a\ pyramid = \frac{area\ of\ the\ base \times height}{3}$. It is applied in other disciplines, such as science, when students study simple machines and learn about the formula *work* = *force* × *distance*. Algebraic reasoning is also used when making decisions in everyday life – for example, when determining which service provider offers a better consumer contract or when calculating how much time it will take for a frozen package to thaw.

Spatial thinking has a fundamental role throughout the Ontario curriculum, from Kindergarten to Grade 12, including in mathematics, the arts, health and physical education, and science. For example, a student demonstrates spatial reasoning when mentally rotating and matching shapes in mathematics, navigating movement through space and time, and using diagonal converging lines to create perspective drawings in visual art and to design and construct objects. In everyday life, there are many applications of spatial reasoning, such as when creating a garden layout or when using a map to navigate the most efficient way of getting from point A to point B.

Algebraic and proportional reasoning and spatial thinking are integral to all STEM disciplines. For example, students may apply problem-solving skills and mathematical modelling through engineering design as they build and test a prototype and design solutions intended to solve complex real-life problems. Consider how skills and understanding that students gain across the strands of the Grade 9 Mathematics course, such as Financial Literacy, Number, and Data, can be integrated into real-life activities. For example, as students collect financial data relating to compound interest, and examine patterns in the data involving compound interest, they apply their understanding of exponents and non-linear growth to generalize rules that can be coded in technology programming environments. This process allows students to create a variety of mathematical models and analyse them quantitatively. These models can then be used to support discussions about what factors can enable or constrain financial decision making, while taking ethical, societal, environmental, and personal considerations into account.

Teaching mathematics as a narrowly defined subject area places limits on the depth of learning that can occur. When teachers work together to develop integrated learning opportunities and highlight cross-curricular connections, students are better able to:

- make connections among the strands of the mathematics curriculum, and between mathematics and other subject areas;
- improve their ability to consider different strategies to solve a problem;
- debate, test, and evaluate whether strategies are effective and efficient;
- apply a range of knowledge and skills to solve problems in mathematics and in their daily experiences and lives.

When students are provided with opportunities to learn mathematics through real-life applications, integrating learning expectations from across the curriculum, they use their lived experiences and knowledge of other subject matter to enhance their learning of and engagement in mathematics. More information can be found in "Cross-Curricular and Integrated Learning".

Literacy in Mathematics

Literacy skills needed for reading and writing in general are essential for the learning of mathematics. To engage in mathematical activities and develop computational fluency, students require the ability to read and write mathematical expressions, to use a variety of literacy strategies to comprehend mathematical text, to use language to analyse, summarize, and record their observations, and to explain their reasoning when solving problems. Mathematical expressions and other mathematical texts are complex and contain a higher density of information than any other text.[10] Reading mathematical text requires literacy strategies that are unique to mathematics.

The learning of mathematics requires students to navigate discipline-specific reading and writing skills; therefore, it is important that mathematics instruction link literacy practices to specific mathematical processes and tasks. To make their thinking visible, students should be encouraged to clearly communicate their mathematical thinking, using the discipline-specific language of mathematics, which provides educators with the opportunity to correct student thinking when necessary.[11] The language of mathematics includes special terminology. To support all students in developing an understanding of mathematical texts, teachers need to explicitly teach mathematical vocabulary, focusing on the many meanings and applications of the terms students may encounter. In mathematics, students are required to use appropriate and correct terminology and are encouraged to use language with care and precision in order to communicate effectively.

More information about the importance of literacy across the curriculum can be found in the "Literacy" and "Mathematical Literacy" subsections of "Cross-curricular and Integrated Learning".

Transferable Skills in Mathematics

The Ontario curriculum emphasizes a set of skills that are critical to all students' ability to thrive in school, in the world beyond school, and in the future. These are known as transferable skills. Educators facilitate students' development of transferable skills across the curriculum, from Kindergarten to Grade 12. They are as follows:

- **Critical Thinking and Problem Solving.** In mathematics, students and educators learn and apply strategies to understand and solve problems flexibly, accurately, and efficiently. They learn to understand and visualize a situation and to use the tools and language of mathematics to reason, make connections to real-life situations, communicate, and justify solutions.
- **Innovation, Creativity, and Entrepreneurship.** In mathematics, students and educators solve problems with curiosity, creativity, and a willingness to take risks. They pose questions, make and test conjectures, and consider problems from different perspectives to generate new learning and apply it to novel situations.
- **Self-Directed Learning.** By reflecting on their own thinking and emotions, students, with the support of educators, can develop perseverance, resourcefulness, resilience, and a sense of self. In mathematics, they initiate new learning, monitor their thinking and their emotions when

[10] Joan M. Kenney et al., *Literacy Strategies for Improving Mathematics Instruction* (Alexandria, VA: Association for Supervision and Curriculum Development, 2005).

[11] William G. Brozo and Sarah Crain, "Writing in Math: A Disciplinary Literacy Approach", *The Clearing House: A Journal of Educational Strategies, Issues and Ideas* 91, no. 7 (2017): 2.

solving problems, and apply strategies to overcome challenges. They perceive mathematics as useful, interesting, and doable, and confidently look for ways to apply their learning.

- **Collaboration.** In mathematics, students and educators engage with others productively, respectfully, and critically in order to better understand ideas and problems, generate solutions, and refine their thinking.
- **Communication.** In mathematics, students and educators use the tools and language of mathematics to describe their thinking and to understand the world. They use mathematical vocabulary, symbols, conventions, and representations to make meaning, express a point of view, and make convincing and compelling arguments in a variety of ways, including multimodally; for example, using combinations of oral, visual, textual, and gestural communication.
- **Global Citizenship and Sustainability.** In mathematics, students and educators recognize and appreciate multiple ways of knowing, doing, and learning, and value different perspectives. They recognize how mathematics is used in all walks of life and how engaged citizens can use it as a tool to raise awareness and generate solutions for various political, environmental, social, and economic issues.
- **Digital Literacy.** In mathematics, students and educators learn to be discerning users of technology. They select when and how to use tools to understand and model real-life situations, predict outcomes, and solve problems, and they assess and evaluate the reasonableness of their results.

Transferable skills can be developed through the effective implementation of high-impact instructional strategies. More information can be found in "Transferable Skills".

Assessment and Evaluation of Student Achievement

Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, First Edition, Covering Grades 1 to 12, 2010 sets out the Ministry of Education's assessment, evaluation, and reporting policy. The policy aims to maintain high standards, improve student learning, and benefit all students, parents, and teachers in elementary and secondary schools across the province. Successful implementation of this policy depends on the professional judgement[12] of teachers at all levels as well as their high expectations of all students, and on their ability to work together and to build trust and confidence among parents and students.

[12] "Professional judgement", as defined in *Growing Success* (p. 152), is "judgement that is informed by professional knowledge of curriculum expectations, context, evidence of learning, methods of instruction and assessment, and the criteria and standards that indicate success in student learning. In professional practice, judgement involves a purposeful and systematic thinking process that evolves in terms of accuracy and insight with ongoing reflection and self-correction".

Major aspects of assessment, evaluation, and reporting policy are summarized in the main "Assessment and Evaluation" section. The key tool for assessment and evaluation in mathematics – the achievement chart – is provided below.

Culturally Responsive and Relevant Assessment and Evaluation in Mathematics

Culturally Responsive and Relevant Pedagogy (CRRP) reflects and affirms students' racial and social identities, languages, and family structures. It involves careful acknowledgement, respect, and understanding of the similarities and differences among students, and between students and teachers, in order to respond effectively to student thinking and promote student learning.

Engaging in assessment from a CRRP stance requires that teachers gain awareness of and interrogate their own beliefs about who a mathematical learner is and what they can achieve (see the questions for consideration provided below). In this process, teachers engage in continual self-reflection – and the critical analysis of various data – to understand and address the ways in which power and privilege affect the assessment and evaluation of student learning. Assessment from a CRRP stance starts with having a deep knowledge of every student and understanding of how they learn best. Teachers seek to build authentic, trusting relationships with students, and with their families and community, as they seek opportunities to build new understanding and support equitable outcomes for all students.

Assessment from a CRRP stance, by its nature, encompasses a wide variety of assessment approaches. It is designed to reflect, affirm, and enhance the multiple ways of knowing and being that students bring to the classroom while maintaining appropriate and high academic expectations for all students. The primary purpose of assessment is to improve student learning. Assessment *for* learning creates opportunities for teachers to intentionally learn about each student and their sociocultural and linguistic background in order to gather a variety of evidence about their learning in an anti-racist, anti-discriminatory environment, in a way that is reflective of and responsive to each student's strengths, experiences, interests, and cultural ways of knowing. Ongoing descriptive feedback and responsive coaching for improvement is essential for improving student learning.

Teachers engage in assessment *as* learning by creating ongoing opportunities for all students to develop their capacity to be confident, independent, autonomous learners who set individual goals, monitor their own progress, determine next steps, and reflect on their thinking and learning in relation to learning goals and curriculum expectations. Teachers engage in culturally responsive and relevant practices by supporting students in the development of these skills by holding positive and affirming views of their students and of their ability to learn and achieve academic success. One way in which teachers differentiate assessment is by providing tasks that allow multiple entry points for all students to engage and that enable all students to access complex mathematics.

Assessment *of* learning is used by the teacher to summarize learning at a given point in time. This summary is used to make judgements about the quality of student learning on the basis of established criteria, to assign a value to represent that quality, and to support the communication of information about achievement to each student, parents, teachers, and others. Teachers engage in culturally responsive and relevant practices that honour and value the importance of student agency and voice in determining the variety of ways in which students can demonstrate their learning.

The evidence that is collected about student learning, including observations and conversations as well as student products, should reflect and affirm the student's lived experiences within their school, home, and community, learning strengths, and mathematical knowledge. This process of triangulating evidence of student learning allows teachers to improve the accuracy of their understanding with respect to how each student is progressing in their learning. Assessment that is rooted in CRRP is an equitable, inclusive, and transparent process that values students as active participants in their learning.

When teachers engage in the process of examining their own biases regarding classroom assessment and evaluation practices, they might consider some of the following questions:

- Are the tasks accessible to, and inclusive of, all learners? Do the tasks include appropriate and varied entry points for all students?
- Do the tasks connect to students' prior learning and give them opportunities to be sense makers and to integrate their new learning? Do the selected tasks reflect students' identities and lived experiences?
- Do all students have equitable access to the tools they need to complete the tasks being set?
- What opportunities can teachers build into their practice to offer students descriptive feedback to enhance learning? Are graded assessment tasks used in a way that complements the use of descriptive feedback for growth?
- How can information be conveyed about students' learning progress to students and parents in an ongoing and meaningful way?
- What is the purpose of assigning and grading a specific task or activity? Are student choice and agency considered?
- How do teacher biases influence decisions about what tasks or activities are chosen for assessment?

The Achievement Chart for Grade 9 Mathematics

The achievement chart identifies four categories of knowledge and skills and four levels of achievement in mathematics. (For important background, see "Content Standards and Performance Standards" in the main Assessment and Evaluation section.)

Knowledge and Understanding – Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Knowledge of content *(e.g., terminology, procedural skills, mathematical models)*	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of content *(e.g., concepts, principles, mathematical structures and processes)*	demonstrates limited understanding of content	demonstrates some understanding of content	demonstrates considerable understanding of content	demonstrates thorough understanding of content
Thinking – The use of critical and creative thinking skills and/or processes				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Use of planning skills *(e.g., understanding the problem; generating ideas; formulating a plan of action; selecting strategies, models, and tools; making conjectures and hypotheses)*	uses planning skills with limited effectiveness	uses planning skills with some effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness

Use of processing skills* *(e.g., carrying out a plan: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at a solution: evaluating reasonableness, making arguments in support of a solution, reasoning, justifying, proving, reflecting)*	uses processing skills with limited effectiveness	uses processing skills with some effectiveness	uses processing skills with considerable effectiveness	uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes* *(e.g., posing and solving problems, critiquing solutions, using mathematical reasoning, evaluating mathematical models, making inferences and testing conjectures and hypotheses)*	uses critical/ creative thinking processes with limited effectiveness	uses critical/ creative thinking processes with some effectiveness	uses critical/ creative thinking processes with considerable effectiveness	uses critical/ creative thinking processes with a high degree of effectiveness
Communication – The conveying of meaning through various forms				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Expression and organization of ideas and information in oral, visual, and/or written forms *(e.g., pictorial, graphic, dynamic, numeric, algebraic forms; gestures and other non-verbal forms; models)*	expresses and organizes ideas and information with limited effectiveness	expresses and organizes ideas and information with some effectiveness	expresses and organizes ideas and information with considerable effectiveness	expresses and organizes ideas and information with a high degree of effectiveness

Communication for different audiences and purposes *(e.g., to share mathematical thinking, to inform, to persuade, to share findings)* **in oral, visual, and/or written forms**	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms *(e.g., terms, symbols, units, labels, structures)*	uses conventions, vocabulary, and terminology with limited effectiveness	uses conventions, vocabulary, and terminology with some effectiveness	uses conventions, vocabulary, and terminology with considerable effectiveness	uses conventions, vocabulary, and terminology with a high degree of effectiveness
Application – The use of knowledge and skills to make connections within and between various contexts				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Application of knowledge and skills *(e.g., selecting and using representations, mathematical tools, and strategies)* **in familiar contexts**	applies knowledge and skills in familiar contexts with limited effectiveness	applies knowledge and skills in familiar contexts with some effectiveness	applies knowledge and skills in familiar contexts with considerable effectiveness	applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills *(e.g., selecting and using representations, mathematical tools, and strategies)* **to new contexts**	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness

Making connections within and between various contexts (*e.g., connections to real-life situations and lived experiences; connections among concepts and representations; connections between mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)*	makes connections within and between various contexts with limited effectiveness	makes connections within and between various contexts with some effectiveness	makes connections within and between various contexts with considerable effectiveness	makes connections within and between various contexts with a high degree of effectiveness

* **Note**: The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* laid out in Strand A: Mathematical Thinking and Making Connections.

Requirements for Strand AA and Strand A

Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics. Learning related to the expectation in Strand AA occurs in the context of learning related to the other six strands. The focus is on intentional instruction; learning in this strand is not included in the assessment, evaluation, or reporting of student achievement.

Strand A: Mathematical Thinking and Making Connections. Strand A has no specific expectations. Students' learning related to this strand takes place in the context of learning related to strands B through F. Student achievement of the expectations in Strand A is to be assessed and evaluated throughout the course.

Criteria and Descriptors for Grade 9 Mathematics

To guide teachers in their assessment and evaluation of student learning, the achievement chart provides "criteria" and "descriptors" within each of the four categories of knowledge and skills.

A set of criteria is identified for each category in the achievement chart. The criteria are subsets of the knowledge and skills that define the category. The criteria identify the aspects of student performance that are assessed and/or evaluated, and they serve as a guide to what teachers look for. In the mathematics curriculum, the criteria for each category are as follows:

Knowledge and Understanding

- knowledge of content (e.g., terminology, procedural skills, mathematical models)
- understanding of content (e.g., concepts, principles, mathematical structures and processes)

Thinking

- use of planning skills (e.g., understanding the problem; generating ideas; formulating a plan of action; selecting strategies, models, and tools; making conjectures and hypotheses)
- use of processing skills (e.g., carrying out a plan: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at a solution: evaluating reasonableness, making arguments in support of a solution, reasoning, justifying, proving, reflecting)
- use of critical/creative thinking processes (e.g., posing and solving problems, critiquing solutions, using mathematical reasoning, evaluating mathematical models, making inferences and testing conjectures and hypotheses)

Communication

- expression and organization of ideas and information in oral, visual, and/or written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; gestures and other non-verbal forms; models)
- communication for different audiences and purposes (e.g., to share mathematical thinking, to inform, to persuade, to share findings) in oral, visual, and/or written forms
- use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms (e.g., terms, symbols, units, labels, structures)

Application

- application of knowledge and skills (e.g., selecting and using representations, mathematical tools, and strategies) in familiar contexts
- transfer of knowledge and skills (e.g., selecting and using representations, mathematical tools, and strategies) to new contexts
- making connections within and between various contexts (e.g., connections to real-life situations and lived experiences; connections among concepts and representations; connections between mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)

"Descriptors" indicate the characteristics of the student's performance, with respect to a particular criterion, on which assessment or evaluation is focused. *Effectiveness* is the descriptor used for each of the criteria in the Thinking, Communication, and Application categories. What constitutes effectiveness in any given performance task will vary with the particular criterion being considered. Assessment of effectiveness may therefore focus on a quality such as appropriateness, clarity, accuracy, precision, logic, relevance, significance, fluency, flexibility, depth, or breadth, as appropriate for the particular criterion.

Mathematics, Grade 9 (MTH1W)

De-streamed

This course enables students to consolidate, and continue to develop, an understanding of mathematical concepts related to number sense and operations, algebra, measurement, geometry, data, probability, and financial literacy. Students will use mathematical processes, mathematical modelling, and coding to make sense of the mathematics they are learning and to apply their understanding to culturally responsive and relevant real-world situations. Students will continue to enhance their mathematical reasoning skills, including proportional reasoning, spatial reasoning, and algebraic reasoning, as they solve problems and communicate their thinking.

Prerequisite: None

Expectations by Strand

Note

Teacher Supports

The expectations are accompanied by “teacher supports”, which may include examples, key concepts, teacher prompts, instructional tips, and/or sample tasks. These elements are intended to promote understanding of the intent of the specific expectations and are offered as illustrations for teachers. *The teacher supports do not set out requirements for student learning; they are optional, not mandatory*.

“Examples” are meant to illustrate the intent of the expectation, the kind of knowledge, concepts, or skills, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails.

“Teacher prompts” are sample guiding questions and considerations that can lead to discussions and promote deeper understanding.

“Instructional tips” are intended to support educators in delivering instruction that facilitates student learning related to the knowledge, concepts, and skills set out in the expectations.

“Sample tasks” are developed to model appropriate practice for the course. They provide possible learning activities for teachers to use with students and illustrate connections between the mathematical knowledge, concepts, and skills. Teachers can choose to draw on the sample tasks that are appropriate for their classrooms, or they may develop their own approaches that reflect a similar level of complexity and high-quality mathematical instruction. Whatever the specific ways in which the requirements outlined in the expectations are implemented in the classroom, they must, wherever possible, be inclusive and reflect the diversity of the student population and the population of the province. When designing inclusive learning tasks, teachers reflect on their own biases and incorporate their deep knowledge of the curriculum, as well as their understanding of the diverse backgrounds, lived experiences, and identities of students. Teachers will notice that some of the sample tasks address the requirements of the expectation they are associated with and incorporate mathematical knowledge, concepts, or skills described in expectations in other strands of the course. Some tasks are cross-curricular in nature and will cover expectations in other disciplines in conjunction with the mathematics expectations.

AA. Social-Emotional Learning (SEL) Skills in Mathematics

Overall Expectation

Throughout this course, in the context of learning related to the other strands, students will:

AA1. Social-Emotional Learning Skills: develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands

Overall expectation

AA1. Social-Emotional Learning Skills

develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands

> This overall expectation is to be included in classroom instruction, but not in assessment, evaluation, or reporting. See further information about approaches to instruction that support all students as they work to apply mathematical thinking, make connections, and develop a healthy identity as mathematics learners to foster well-being and the ability to learn mathematics.

Teacher supports

Examples

The following examples illustrate various ways teachers may provide instruction to support students in developing social-emotional learning skills in connection to learning mathematics.

Recognizing and Identifying Emotions That Support Mathematical Learning

Scenario [checking observations with student]: A student has been given the task of multiplying powers with variable bases (e.g., $a^2 \cdot a^8$).

Teacher Observation: The teacher perceives that the student appears to be frustrated with the task.

Note: Each teacher's perspectives are subject to and informed by their own experiences. What the teacher observes and perceives may or may not align with what the student is actually feeling and experiencing. A culturally responsive and relevant approach starts with the teacher engaging in self-reflection, then considering elements of an inclusive learning environment and the educational context in which they are observing the student.

Teacher Action and Student Response: The teacher asks the student if they would like to talk through the task together. The student accepts. During the conversation, the teacher supports the student in identifying the emotions they are feeling and helps build the student's understanding that thoughts, feelings, and actions are all connected and affect one another. In this case, the student identifies that they are feeling confused and frustrated. The teacher works with the student to identify strategies that will help in this situation (e.g., identify what they do understand, develop related mathematical literacy, make connections by relating the task to what they know about powers with numeric bases, seek further information by reviewing class notes about exponent laws, explore different ways of looking at the problem, take a break and come back to the task later). In this case, the teacher asks the student to think about a similar situation and strategies they may have used when they multiplied powers with bases that were integers. For example, the student could be encouraged to write out the expanded form of the expression, without exponents, use that expression to recall what to do and why the method makes sense, then generalize this understanding and apply the method to the problem with the variable bases. In this way, the student makes the connection to their prior learning and applies it to complete the task. The student could then reflect on how the approach they took to work through this task might help them the next time they feel frustrated with a task.

Teacher Reflection [continue reflection on an ongoing basis]: The teacher reflects on how their actions may have affected the student's level of confidence in using strategies for continuing to problem solve when the student feels frustrated. The teacher thinks about potential alternatives for future interactions with this student. This interaction also supports teacher decisions on future tasks that may improve this particular student's perseverance skills and confidence.

Note: Ongoing teacher reflection is important throughout instruction, not just at the beginning and end. It includes developing an understanding of individual student identities, strengths, and needs, including language learning and educational experience. This is a critical first step in building trust and relationships with students.

Recognizing Sources of Stress That Present Challenges to Mathematical Learning

Scenario [using culturally responsive pedagogy to build a plan of action in response to individual student strengths and needs]: A student has been given a task that involves fractions.

Student's Initial Reaction: The student shares with the teacher that they get stressed every time they encounter a fraction.

Teacher and Student Conversation: The teacher thanks the student for sharing information about how they are feeling and asks if they would like to share more about why fractions create a stressful response for them. The student shares with the teacher that they do not see the relevance of fractions to their everyday life, and that they have always struggled with fractions for as long as they can remember.

The teacher acknowledges the student's feelings of stress, and helps the student respond to these feelings by identifying what they do know about fractions. Next steps could include identifying what additional support is needed to help the student feel less stressed and be more successful and confident when working with fractions. For example, the teacher could select mathematical tasks that are contextualized to provide a relatable entry point for this student, including the use of fraction manipulatives (e.g., fraction strips, relational rods) to complete the tasks.

Teacher Reflection [consider strengths and needs of individual students]: The teacher thinks about how their approach to working with individual students to identify personal strategies to respond to stress seemed to have been received by this particular student. This provides useful information about strategies that might best support this student in the future.

Identifying Resources and Supports That Aid Perseverance in Mathematical Learning

Scenario [providing one-on-one support]: A student has been given the task of writing code to explore what happens to the volume of a rectangular prism when one, two, and three dimensions of the prism are altered.

Student's Initial Reaction: The student shares with the teacher that they do not know how to begin the task and they feel overwhelmed.

Teacher's Response to Student: The teacher has a conversation with the student to learn about their prior experience with coding. The teacher then works directly with the student, supporting them as they develop a physical model and a flow chart to plan out the code for exploring what happens to the volume of a rectangular prism when one dimension changes. Next, the teacher has the student write the code and execute it to see if they get the output they were expecting.

Student's Response to Teacher: The student shares that the code they wrote produced what they expected.

Teacher's Response to Student: The teacher then works with the student to identify how the model and the flow chart will need to be altered to reflect a change in two dimensions instead of one. The teacher points out that knowing the steps to follow and knowing that it is okay if the results aren't as expected both aid in persevering with the task.

Student's Response to Teacher: The student shares with the teacher that they now know what to do and can proceed with writing the remaining code to complete the task.

After getting to this stage, the student reflects that one of the benefits of coding is that one can get feedback right away, and that it is okay if the results are not as expected. Identifying the reason for the difference is part of the process and the challenge.

Teacher Reflection [consider strategies for uplifting students and impact of this]: The teacher thinks about aspects of what this student did that could be helpful to other students. After inquiring whether this student is willing to share their successes, the teacher can proceed to highlight for the class effective strategies that this student employed. This might then further improve this student's experience with coding as well as their confidence. The teacher can subsequently reflect on how these interactions were helpful for this student as well as for their peers and imagine ways the experience could have been improved.

Building Healthy Relationships and Communicating Effectively in Mathematics

Scenario [supporting individual students within a group setting]: The teacher is presenting a mathematical modelling task that students are to complete in a small group.

Students' Initial Reaction: Some students express discomfort with working in a small group.

Teacher Prompt: The teacher checks in with these students to find out what makes them feel uncomfortable about working in a group. The students indicate that the lack of structure and accountability in group discussions and settings bothers them and makes them nervous about completing the task. Students may need support to describe what makes them uncomfortable.

Whole-Class Discussion: The teacher facilitates a discussion with the class to co-create working agreements that include guidelines and practices that are important to them in order to collaborate effectively as a group – providing brief examples of effective strategies that they already have seen from the students. Guidelines and practices could include listening attentively to each other's ideas; having one person at a time share their idea; ensuring that all ideas are respected and valued; deciding as a group which ideas for solving the task they will explore further. Students describe what these co-constructed agreements may look and sound like; for example, listening to ideas before commenting, or asking probing questions to clarify. The teacher records the agreements and posts them in the classroom as a reference for all group tasks.

Small-Group Discussion: The students begin working in small groups to complete the mathematical modelling task. The teacher visits each of the groups to monitor whether discussions are following the working agreements that were set by the class. The teacher checks in privately with the students who had expressed discomfort in group settings to see how they are feeling and identifies whether additional supports are needed in order for the students to be able to interact meaningfully with the group discussion. The teacher facilitates the discussion with each group by posing questions about the mathematical modelling task and supports students in using the guidelines that have been set.

Teacher Reflection [inform future action]: The teacher considers how these students responded to the discussion, and how future activities can be structured with a positive experience for these students in mind. The teacher reflects on how the guidelines established by the class can be revisited and further established in future conversations with students – recognizing that this is a fluid process.

Developing a Healthy Mathematical Identity Through Building Self-Awareness

Scenario [building empowerment through relevance using a strength-based approach]: Students are given the task of researching and telling a story about the development of a geometric concept that is relevant to them. The teacher provides guidance for a student who is unsure of how to approach this.

Small-Group Discussion: The teacher asks students to brainstorm in small groups some possible geometric concepts that they might be interested in researching and identify why each concept is of interest.

Teacher Prompt: The teacher asks students to select one of the concepts from their brainstorming session that might be relevant to them, in that they can make a personal connection to the concept or tell a story about it in a way that reflects their identities or interests.

Students' Action: Students gather information on their selected concepts. They may collect information from personal history, community organizations, or other resources.

Teacher Prompt: The teacher prompts students to decide how they would like to create and share their story about their chosen concept with others in the class. The teacher also asks the students to make connections to a career or to a discipline such as the arts (visual or media arts or dance).

Student and Teacher Discussion: For the student who was unsure of how to create and share their story, the teacher and student work together to identify an approach that uses the student's strengths and highlights the relevance of the concept they have chosen (e.g., telling their story through a musical composition or a visual display).

Teacher Reflection [connect the intent of the curriculum and various global perspectives]: The teacher reflects on how stories of geometric concepts were represented by various cultures demonstrates their significant contributions to mathematics. The teacher thinks about including the stories of various cultures in their teaching so that students come to understand and appreciate mathematics as a human story.

Developing Critical and Creative Mathematical Thinking

Scenario [supporting students who are working in pairs in developing an appreciation of other perspectives]: Students are given the task of creating a table of values, a graph, and an algebraic expression to represent a linear relation.

Students' Actions: One student chooses to start with creating a graph to represent their relation. Another student chooses to start with creating a table of values.

Teacher Facilitation: The teacher is monitoring the class as students are creating their representations and notices that two students are approaching the task by starting with different representations. The teacher checks with the students to see if they are okay with sharing with the rest of the class their work so far. The students agree, and the teacher pauses the class and invites the students to share. The teacher reinforces the idea that mathematicians use different representations as starting points in their problem solving and consolidates the critical understanding that different starting points and approaches can lead to the same result. These two students share their reasoning for why they chose to start with the representations that they did, and elaborate on how they made connections and decisions as part of their thinking process.

Students' Responses: The student who started with a graph shared that they like to see whether they are creating a line that has a positive slope or a negative slope. For them, a visual representation is a helpful starting point. The student who started with the table of values shared that they wanted to create their linear relation by seeing the pattern between each pair of points. For this student, being able to see the numbers side by side helps to show the relationship between the numbers.

Teacher Facilitation: The teacher encourages the rest of the students to think about why they chose the representation that they did, and then asks them to also think about the benefits of being aware of the reasons behind their decisions. The teacher also prompts a discussion about the connections among the different representations and how each of them provides information, such as comparing the amount the graph goes up or down at each iteration with the change in the value in the chart of the dependent variable. Next, this might also be compared to the slope, and the meanings all explicitly connected.

Teacher Reflection [consider the effectiveness of messaging]: The teacher reflects on the effectiveness of their comments about various representations – whether students connected their own experiences with the summary that the teacher shared when facilitating the discussion. The teacher may continue to reflect on the direction of future lessons, and on whether more activities are necessary or helpful for students to better appreciate the importance of connecting various representations.

Instructional Tips

Approaches to Instruction of Social-Emotional Learning Skills

See the section Elements of the Grade 9 Mathematics Course – Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics for essential information on approaches to instruction.

Opportunities for Planning Instruction

Teachers are encouraged to look for opportunities to highlight and embed explorations of and connections to social-emotional learning skills, where appropriate, within the learning throughout the course, in order to support students in developing and applying these skills.

When reviewing the **Instructional Tips** provided in each strand and planning for instruction throughout the course, teachers consider how to use strategies such as the sample strategies outlined in the section mentioned above to support the instruction of social-emotional learning skills in an inclusive way.

Below are some examples of opportunities that teachers might find for instruction of social-emotional learning skills as part of students' learning related to the other expectations throughout the course. Note that that there are many other possible opportunities for this to occur, taking into account students' strengths and needs and the learning context.

- **Recognizing and Identifying Emotions That Support Mathematical Learning**
  Through instruction, teachers can support students in:
  - recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort, and honouring those emotions as they arise; for example, when students are writing and altering code to represent mathematical situations (C2.1, C2.2).

Note

Engaging in self-reflection is key for teachers in recognizing that various experiences in mathematics will elicit a range of emotions from students. It is important for teachers to be responsive to emotions they observe in the students they are working with and work to avoid anticipating certain emotions based on personal experience. See Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics for further guidance.

- **Recognizing Sources of Stress That Present Challenges to Mathematical Learning**
  Through instruction, teachers can support students in:
  - approaching peers, teachers, other staff, family, and/or their extended community for support in a range of situations; for example, when students are applying the process of mathematical modelling to solve real-life problems (D2.2, D2.3, D2.4, D2.5);
  - applying strategies such as engaging in guided imagery and visualization to help make mathematical connections; for example, when students are making connections between $y = ax$ and its various transformations (C4.3).
- **Identifying Resources and Supports That Aid Perseverance in Mathematical Learning**
  Through instruction, teachers can support students in:
  - recognizing mistakes as a necessary and helpful part of learning; for example, when students are solving problems involving conversions between different units or between measurement systems (E1.3);
  - encouraging students to persevere and seek support when they find concepts and exercises to be challenging;
  - noticing strengths and positive aspects of experiences, appreciating the value of practice and the necessity of repetition, and reflecting on the process of practice; for example, when students are solving problems involving operations with positive and negative fractions and mixed numbers (B3.4).
- **Building Healthy Relationships and Communicating Effectively in Mathematics**
  Through instruction, teachers can support students in:
  - listening attentively and respectfully in various situations; for example, when peers are sharing their stories about a mathematical concept of their interest, in order to understand

    and appreciate the perspectives of others' identities, knowledge, and experiences (B1.1, C1.1, E1.1);
  - considering other ideas and perspectives from peers, parents, and the wider community; for example, when students are sharing their rationales for budget modifications (F1.4).
- **Developing a Healthy Mathematical Identity Through Building Self-Awareness**
  Through instruction, teachers can support students in:
  - identifying their personal strengths and exercising their own creativity as they engage in a variety of tasks; for example, when students are creating and analysing geometric designs that are relevant to them (E1.2);
  - nurturing a sense of belonging and community; for example, when students are making connections between mathematics, various knowledge systems, and real-life applications of mathematics, by recognizing a range of experiences (A2).
- **Developing Critical and Creative Mathematical Thinking**
  Through instruction, teachers can support students in:
  - making connections between different forms of representation; for example, when students are comparing characteristics of graphs, tables of values, and equations of linear and non-linear relations (C4.1);
  - making decisions; for example, when students are considering how to represent and analyse data (D1.2).

A. Mathematical Thinking and Making Connections

This strand has no specific expectations. Students' learning related to this strand takes place in the context of learning related to strands B through F, and it should be assessed and evaluated within these contexts.

Overall Expectations

Throughout this course, in connection with the learning in the other strands, students will:

A1. Mathematical Processes: apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

A2. Making Connections: make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

Overall expectation

A1. Mathematical Processes

apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

Teacher supports

Instructional Tips

Teachers can:

- highlight the interconnectedness of the seven mathematical processes (problem solving, reasoning and proving, reflecting, connecting, communicating, representing, and selecting tools and strategies) and model ways for students to combine them while doing mathematics;
- support students in activating their prior knowledge when encountering new concepts *(connecting)*;
- provide students with opportunities to integrate their learning within and across the strands, explicitly demonstrating and reinforcing connections between various mathematical concepts *(connecting)*;
- support students' appreciation of the innate beauty of mathematics *(problem solving, reasoning and proving, reflecting, connecting, communicating, representing, selecting tools and strategies)*;

- pose problems that have multiple entry points and can be solved in various ways *(problem solving)*;
- provide students with opportunities to pose and solve authentic problems that are of interest to them *(problem solving)*;
- make available a range of materials and technologies for students to choose from and teach them how to select and use tools to represent mathematical situations, solve problems, and communicate their thinking *(selecting tools and strategies)*;
- support students in understanding that the same mathematical situation can be represented in various ways, and in making connections among different representations *(representing, connecting)*;
- empower students to think about and strategize how they solve problems, including steps such as representing the situations, selecting tools and strategies, reflecting on the reasonableness of their solutions, and justifying their thinking *(problem solving, representing, selecting tools and strategies, reflecting, reasoning and proving)*;
- encourage students to reflect on their mistakes and on feedback they are given and to revise their mathematical solutions as necessary, demonstrating how doing so can move learning forward *(reflecting)*;
- provide opportunities for students to respectfully listen, reflect, and discuss strategies and reasoning in pairs or small groups *(communicating, reflecting, reasoning and proving)*;
- facilitate the purposeful sharing of different problem-solving strategies for the same problem, including validating, recognizing, and encouraging fruitful aspects in each student's strategy *(communicating, reflecting)*;
- support all students in expanding their communicative repertoire to include a broader range of terminology and conventions *(communicating)*;
- create opportunities for meaningful peer feedback and emphasize the benefits of discourse in the learning of mathematics *(communicating)*.

Teacher Prompts

Prompts that highlight specific mathematical processes

Problem Solving

- Explain in your own words the problem you need to solve.
- What information, knowledge, and strategies may be helpful to solve this problem?
- What assumptions are inherent in the problem? What assumptions are you making?
- What information do you know already and what additional information is required to solve the problem?

Reasoning and Proving

- Is this statement true for all cases?
- How can you verify this answer?
- What would happen if …? (e.g., if the rate of change increased? if this number were negative?)
- How can you extend these ideas to more general cases?

Reflecting

- Does this answer make sense to you? Why or why not?
- How did the learning tool you chose contribute to your understanding of, or solution for, the problem?
- How did the tool assist you in communicating your answer or your thinking?
- What strategies did you use that did or did not work?
- What did you learn in the process of working through this problem?

Connecting

- What connection(s) do you see between a problem you solved previously [describe the problem student solved previously] and today's problem?
- Describe the connections you see between … and …. (e.g., one representation with another, a student's interpretation with another, a student's strategy with another)
- How does your representation (e.g., diagram, sketch, concrete/digital representation) connect to …? (e.g., the algebraic solution? another student's work?)
- How is the strategy that was just shared in the group discussion similar to or different from your strategy?

Communicating

- Present your solution to a problem so that someone else will understand your thinking and your process.
- Share your thinking with this group and consider their feedback as you revise your work.
- How can you express this in another way?

Representing

- In what other way(s) can you represent this situation?

- How could you represent this situation using a graph? using a diagram? using concrete/digital tools? using a table of values?
- In what way(s) would a scale model help you solve this problem?

Selecting Tools and Strategies

- Explain why you chose to use this tool/strategy to solve the problem.
- What other tools/strategies did you consider using? Explain why you chose not to use them.
- What were the advantages and disadvantages of the strategies you tried?
- What estimation strategy did you use? Was your result sufficiently accurate for the situation?

Sample Tasks

Sample tasks from Strands B to F that highlight various mathematical processes:

Number

B2.1 Sample task that highlights reasoning and proving, and selecting tools and strategies:

- Have students determine, using a strategy of their choice, which number is smaller, -4×10^3 or 4×10^{-3}.

Algebra

C4.2 Sample task that highlights representing and making connections:

- Ask students to generate a set of coordinates that satisfy the equation $x + y = 10$. Have them plot these coordinates on a grid, and then discuss whether they have found all the possible values that satisfy the equation or if there are others between these points. This discussion should lead to the idea of connecting the points with a line to represent all possible values. Then have them choose points above and below the line they have drawn, and ask them how these points are connected to the inequalities $x + y > 10$ and $x + y < 10$.

Data

D2.5 Teacher prompts that highlight reflecting (after completing a mathematical modelling task):

- Does your model help you to answer your question? Did you need to revise the model? Why?

- Does your model allow you to make predictions?
- What predictions can be made based on the model?
- What are the limits of the model?

Geometry and Measurement

E1.3 Sample task that highlights problem solving:

- Have students solve problems that involve using unconventional units to measure. For example: How many of the same type of coin are needed to go around the circumference of Earth?

Financial Literacy

F1.4 Sample task that highlights communication:

- Show students the budget for a division of the local municipal government (e.g., Parks and Recreation). Pose a scenario that is relevant to the current local situation (e.g., community members would like an outdoor skating rink) and have students discuss how the budget could be modified based on this scenario.

Overall expectation

A2. Making Connections

make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

Teacher supports

Instructional Tips

Teachers can:

- demonstrate that they honour and value connections identified by students, and share their excitement over the connections they discover;
- build on students' sense of their identities, experiences in school and other experiences, ideas, questions, and interests to support the development of an engaging and inclusive mathematics learning community;
- create opportunities for every student to feel that they are reflected in mathematical learning;

- facilitate discussions about the applications of mathematics in the world outside the classroom, including the natural world;
- provide opportunities for students to recognize that mathematical knowledge has been developed by every culture around the globe;
- respectfully incorporate, in partnership through community connections (e.g., a community member, Elder, knowledge holder, or other individual with expertise), specific examples that highlight First Nations, Métis, and Inuit cultures and ways of being and knowing, in order to infuse Indigenous knowledges and perspectives meaningfully and authentically into the mathematics program;
- facilitate student-generated class discussions about careers and fields of study that students are interested in exploring.

Note

Teachers are encouraged to collaborate with community partners to plan culturally responsive and relevant teaching that honours and respects students' identities, and that includes real-life applications of mathematics that are relevant to students' lives and their communities.

Teacher Prompts

- How does this mathematical concept connect to a story that has been shared by others in the class?
- How does this mathematical concept connect to something you have experienced in your life?
- How do you see this mathematical concept applied in real life?
- What are some careers that may use this mathematical concept, and in what ways?

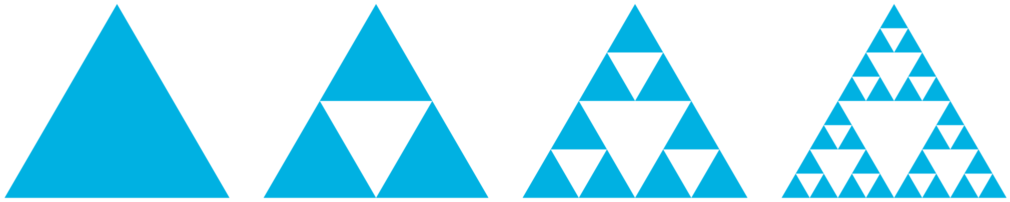
Sample Tasks

Connections students may make when completing sample tasks for strands B to F:

Number

B1.3 Sample Task

Have students make their own fractal triangles (Sierpinski triangles) by repeating a simple pattern of triangles to create a complex image, then use these images to create as many kinds of different number sets as they can (e.g., the number of blue triangles in each image, the fraction of blue triangles to white triangles in each image) or another similar fractal.



A student engaged in this task might make connections between the concept of infinity in their fractal designs and the fractal designs they have seen in nature, such as in pine cones, ice crystals, and trees.

Algebra

C3.2 Sample Task

Provide students with one representation of a linear relation in context, and ask them to create a different representation of the relation. Some examples of contexts that might be relevant to students' lives are:

- cost of participating in various classes (e.g., dance, yoga, martial arts, fitness, music)
- distance travelled over time
- number of hours worked and total pay
- mass of bulk goods purchased and cost
- area of land and crop yield

A student engaged in this task might make connections between a real-life context that is familiar to them and the way each type of representation reveals information about that context.

Data

D2.1 Sample Task

Have students research careers that involve mathematical modelling.

A student engaged in this task might make connections between mathematical modelling and careers such as game designer, meteorologist, actuary, marketing analyst, and biostatistician.

Geometry and Measurement

E1.3 Sample Task

Ask students to share a recipe that is relevant to them, their family, and/or their community. Have them exchange recipes and pose questions for other students to answer, such as:

- What ingredient do you need the most of, and how can you tell?
- If you are missing one of the measuring tools, how can you use another measuring tool to measure the appropriate amount?
- If the recipe uses mass, how can you convert it to use capacity measuring tools?

A student engaged in this task might make connections between recipes that use imperial measures, such as cups, those that use metric measures, such as grams, and other types of measures, such as handfuls.

Financial Literacy

F1.2 Sample Task

Have students brainstorm examples of assets that appreciate or depreciate, including those that might experience a short-term appreciation due to a current trend (e.g., trading cards, trends started on social media). Show students graphs or provide them with data to graph depicting the appreciation and depreciation of assets identified during the brainstorming session and have them identify what they notice and what questions they might still have about each of the graphs.

A student engaged in this task might make connections between assets that appreciate or depreciate in value and items they are collecting, such as video games, comic books, or sports memorabilia.

B. Number

Overall Expectations

By the end of this course, students will:

B1. Development of Numbers and Number Sets: demonstrate an understanding of the development and use of numbers, and make connections between sets of numbers

B2. Powers: represent numbers in various ways, evaluate powers, and simplify expressions by using the relationships between powers and their exponents

B3. Number Sense and Operations: apply an understanding of rational numbers, ratios, rates, percentages, and proportions, in various mathematical contexts, and to solve problems

Overall expectation

B1. Development of Numbers and Number Sets

demonstrate an understanding of the development and use of numbers, and make connections between sets of numbers

Specific expectations

By the end of this course, students will:

B1.1 Development and Use of Numbers

research a number concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context

Teacher supports

Examples

- **stories that students may share involving number concepts:**
  - number systems:
    - The Chinese number rod system, dating from as far back as 200 BCE, used red to represent positive numbers and black to indicate negative numbers. This number system was used in commerce to indicate that the amount sold was positive and the amount spent was negative.
  - multiplicative thinking and proportional reasoning:
    - These concepts are used in many cultures and communities in beadwork and embroidery to produce items with repeating patterns of a certain size.

- golden ratio (phi):
  - The golden ratio is a ratio of length to width, approximately equal to 1.618 : 1, and is considered to be a visually appealing proportion. This ratio can be seen in nature in the spiral patterns on pine cones, fruits, and vegetables. The golden ratio is also evident in art, design, and architecture. For example, the CN Tower uses the golden ratio in its design.

Instructional Tips

Teachers can:

- on an ongoing basis, both formally and informally, encourage students to bring into the classroom real-life stories they have gathered about the mathematical concepts they are learning, in order to enhance their understanding of these concepts and make connections between them and with ordinary life;
- build an authentic and inclusive learning environment where students are encouraged to learn about knowledge systems from around the world, including Indigenous ways of knowing.

Note

Students can seek out real-life stories through conversations with people in their families or communities, or through print and digital resources. They may need guidance on seeking information about new perspectives on mathematics. The aspect of student choice in this expectation may also involve the teacher taking the stance of a co-learner as they support students in exploring stories of various cultures.

Teacher Prompts

- What do you think a number concept is?
- What constitutes a numeration system? How did various numeration systems come to be?
- How did the place-value system come to be?
- What are some interesting numbers you have seen? Why are they interesting to you?
- Did you begin your research with a specific concept or a specific culture in mind?
- What did you find out about the development and use of this number concept in a specific culture?
- How might this concept be relevant to you? To our learning today?
- How is this number concept used across various disciplines?
- How did various cultures contribute to the development of this number concept? Was there a loss of cultural knowledge due to these developments?

Sample Tasks

1. Have students brainstorm possible number concepts that they could research. Ask them to choose a concept of interest from the list and gather information about its socio-historic development in a culture of their choice. Once students have gathered their information, have them decide how they would like to tell the story of the development of the concept to the class.

2. Have students collaborate on creating a timeline for the stories they find.

3. Have students work in groups or as a whole class to create a world map that shows the connection of their number concepts to a culture. Invite students to share their observations and reflections on the relevance of those concepts to their own current context.

4. Have students consider traditional border patterns or trim designs on parkas or other clothing. Have them describe the number concepts they see applied or represented in the pattern or design.

B1.2 Number Sets

describe how various subsets of a number system are defined, and describe similarities and differences between these subsets

Teacher supports

Examples

- **subsets of an identified number system:**
  - odd and even numbers are subsets of integers
  - prime and composite numbers are subsets of integers
  - rational and irrational numbers are subsets of real numbers
  - triangular numbers is a subset of whole numbers
  - perfect squares is a subset of whole numbers

Instructional Tips

Teachers can:

- highlight that every number system is made up of a set of numbers as well as certain arithmetic operations we define on them;
- create learning tasks that enable students to identify unique characteristics of various subsets using visual and concrete representations, such as representing perfect squares with square tiles to show that perfect squares can be represented in the shape of a square, that they are the result of multiplying an integer by itself, and end with only digits 0, 1, 4, 6, 9, or 25;
- discuss the idea that subsets are defined by people, and may be created for specific purposes;

- incorporate coding to reinforce students' understanding of connections between various number systems and types of numbers within a system.

Teacher Prompts

- What is a number system?
- How can we describe what it means to belong in a given number set?
- What are some words we can use to describe number sets? Can you explain in your own words what prime numbers are? composite numbers? rational numbers?
- A certain subset contains the number 5. What are some possible characteristics of that subset other than that it contains 5?
- Why is every second multiple of 6 also a multiple of 4?

Sample Tasks

1. Have students write code to generate the first 100 prime numbers or the prime numbers between a specific interval (e.g., 700–800).

2. Have students create a number set and then pair up. Have partners look at the numbers in each other's sets and describe the characteristics of the set.

3. Have students describe similarities and differences between different subsets and discuss how the same numbers can belong to different subsets.

4. Have students create a diagram to show the similarities and differences between number sets: natural numbers, whole numbers, integers, rational numbers, and real numbers.

5. Have students create subsets of numbers that have different amounts of numbers in common: for example, no numbers in common, a finite number of numbers in common, an infinite number of numbers in common.

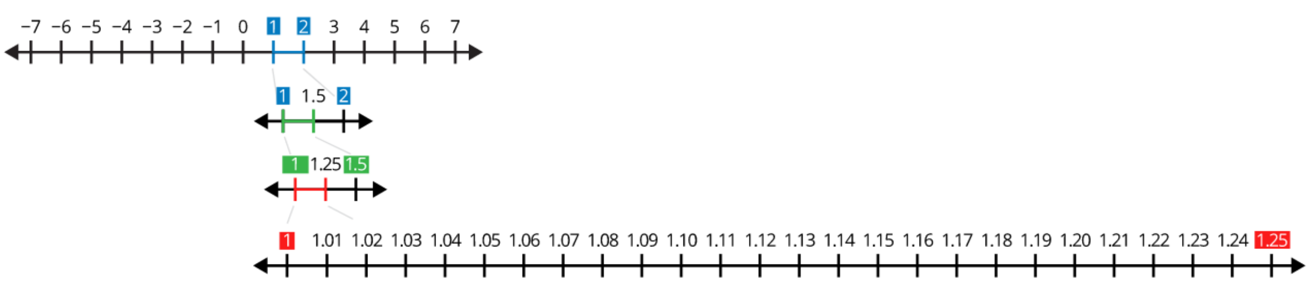
B1.3 Number Sets

use patterns and number relationships to explain density, infinity, and limit as they relate to number sets

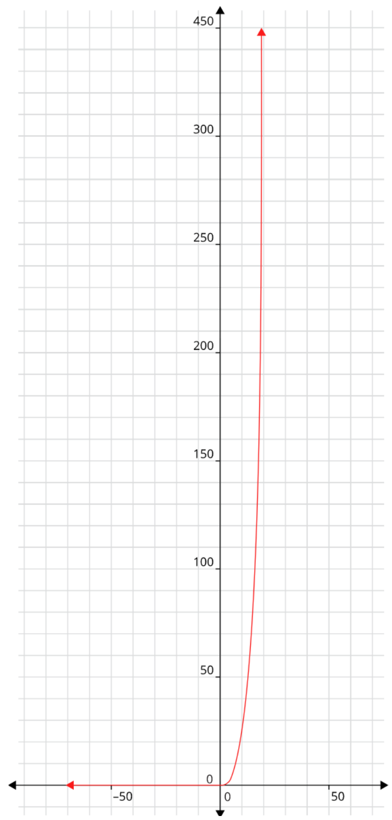
Teacher supports

Examples

- **patterns and number relationships:**
    - growing patterns can be extended indefinitely:
        - 2, 4, 6, 8, …
        - −5, −3, −1, 1, …
        - 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$
        - 2, 4, 8, 16, …
        - 1, 4, 9, 16, …
- explaining the density of a number set as the probability that a number of a particular type will be found within a particular set of numbers:
    - the set of perfect squares between 0 and 20 is less dense than the set of odd numbers between 0 and 20 because the respective probabilities are $\frac{5}{21}$ and $\frac{10}{21}$
    - the set of real numbers between −2 and 6 is more dense than the set of integers between −2 and 6 because there is an infinite number of real numbers in that set, but only 7 integers
- exploring the density and infinity of a number set:
    - using a number line:



○ using a graph:



○ using a table to show a number relationship where the values seem to approach infinity:

x	**2^x**
0	1
1	2
2	4
10	1024
20	1 048 576
30	1 073 741 824
. . .	. . .

- using a table to show a number relationship where the values approach a limit, in this case 0:

x	2^x
1	2
0	1
−1	0.5
−2	0.25
−5	0.031 25
−8	0.003 906 25
−10	0.000 976 562 5
. . .	. . .

Instructional Tips

Teachers can:

- begin the learning with familiar patterns and number relationships, such as odd and even numbers and multiples of 25, and familiar strategies, such as skip counting and using tiles, then support students in making connections between number patterns and the learning in C1.1 about generalizing the number relationships using algebraic expressions;
- highlight that pattern rules are a way of generalizing a number relationship so it can be extended indefinitely;
- support students in distinguishing between cardinality of finite sets, which identifies the number of elements in a set, and density, which refers to the likelihood that any given number in the set is of a particular type;
- focus students' attention on the notion that a set's density is described as relative to another subset, using words such as "less than" and "greater than" (e.g., the set of rational numbers is less dense than the set of real numbers);
- create opportunities for students to use coding to extend their thinking;
- support students in developing an understanding of the numbers situated within a given interval, while making connections to learning in elementary about the notation of "…" (ellipsis) at the end of a pattern and the arrows at the end(s) of a number line to indicate that the pattern can continue indefinitely;

- discuss real-life examples, such as fractals, to highlight the concept of infinity, and make connections to learning in geometry and measurement to illustrate this concept in a variety of ways;
- elaborate on the ways in which various cultures have understood the concept of infinity.

Teacher Prompts

- Explain how you know that there are an infinite number of fractions between $\frac{8}{10}$ and $\frac{9}{10}$.
- How might a pattern continue if it begins with 1, 4, …?
- Is the number set created from your pattern finite or infinite? Compare your number set with that of a partner. Do they have the same density? Do they both have a limit?
- Are there more triangular numbers or more prime numbers between 1 and 100? How many more?
- What do you observe about the size of each successive number in the set: $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$?

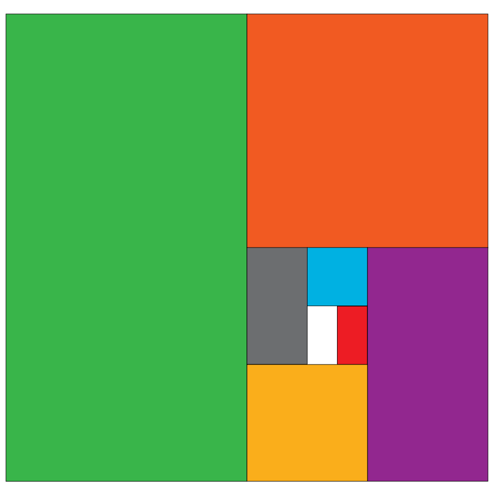
Sample Tasks

1. Have students compare the densities of the set of even numbers between 0 and 100 (inclusive) and the set of real numbers between 0 and 100 (inclusive).

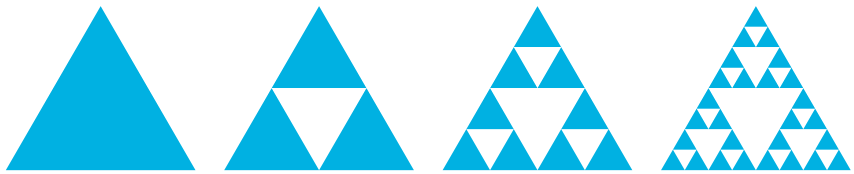
2. Have students create visuals or patterns of numbers that can be extended indefinitely.

3. Have students come up with a pattern of numbers in which each number gets progressively closer to the number 2.

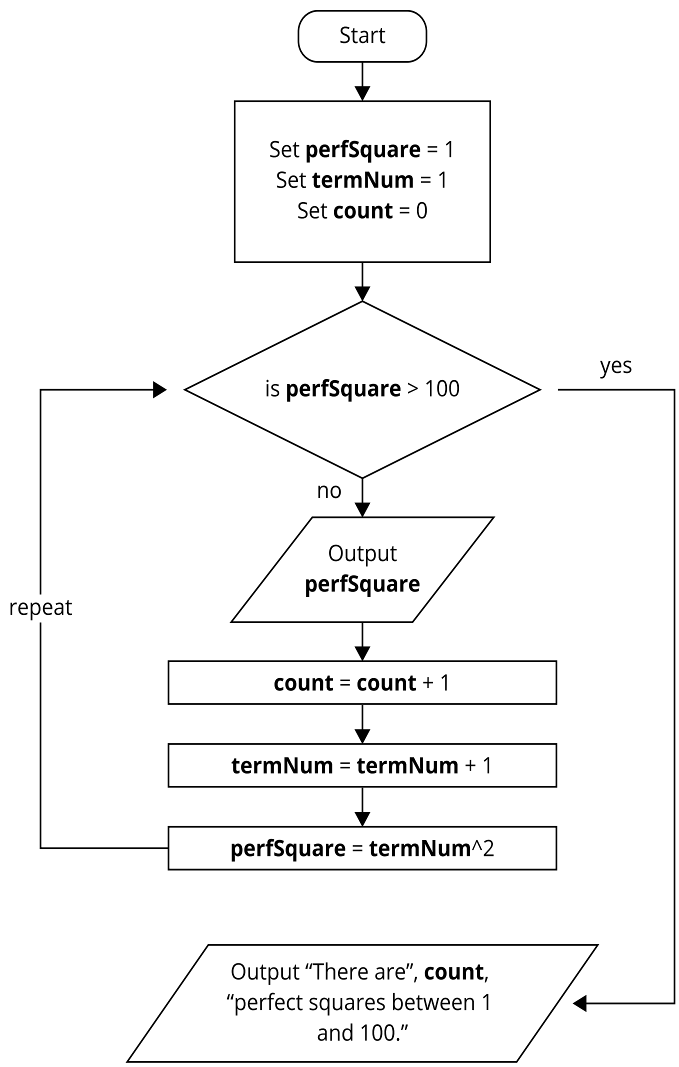
4. Have students describe the pattern they observe in the coloured sections of the square below and make predictions about how much of the total square will eventually be coloured.



5. Have students make their own fractal triangles (Sierpinski triangles) by repeating a simple pattern of triangles to create a complex image, then use these images to create as many kinds of different number sets as they can (e.g., the number of blue triangles in each image, the fraction of blue triangles to white triangles in each image) or another similar fractal.



6. Have students alter the following flow chart to determine the number of perfect squares between 0 and 1000. Have students write and execute the code for the given flow chart and the altered flow chart and then use the outcomes to describe the density of the set of perfect squares between 0 and 100 compared to the density of the set of perfect squares between 0 and 1000.



Overall expectation

B2. Powers

represent numbers in various ways, evaluate powers, and simplify expressions by using the relationships between powers and their exponents

Specific expectations

By the end of this course, students will:

B2.1 Powers

analyse, through the use of patterning, the relationship between the sign and size of an exponent and the value of a power, and use this relationship to express numbers in scientific notation and evaluate powers

Teacher supports

Examples

- **relationship between exponent of a power and the value of the power:**
    - patterns can be used to represent relationships among numbers expressed as powers
        - as illustrated in the chart below, when the size of an exponent decreases by 1, the value of the power is divided by the base, in this case, 2:

	Power	Expands to	Value	
	2^3	$2 \times 2 \times 2$	8	
$\div 2$	2^2	2×2	4	$\div 2$
$\div 2$	2^1	2	2	$\div 2$
$\div 2$	2^0	$\frac{2}{2}$	1	$\div 2$
$\div 2$	2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$	$\div 2$
$\div 2$	2^{-2}	$\frac{1}{2 \times 2}$	$\frac{1}{4}$	$\div 2$

- as illustrated in the chart below, when the size of an exponent decreases by 1, the value of the power is divided by the base, in this case, *a*:

	Power	Expands to	Value	
	a^3	$a \times a \times a$	a^3	
$\div a$	a^2	$a \times a$	a^2	$\div a$
$\div a$	a^1	a	a	$\div a$
$\div a$	a^0	$\frac{a}{a}$	1	$\div a$
$\div a$	a^{-1}	$\frac{1}{a}$	$\frac{1}{a}$	$\div a$
$\div a$	a^{-2}	$\frac{1}{a \times a}$	$\frac{1}{a^2}$	$\div a$

- **numbers expressed in scientific notation**:
  - a DNA strand has a width of about 2.5×10^{-9} m:
    $0.000\,000\,002\,5 = 2.5 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$
    $= 2.5 \times 10^{-9}$ m
  - $-4\,000\,000\,000 = -4.0 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
    $= -4.0 \times 10^9$
- **evaluating powers**:
  - powers expressed with rational number bases and integer exponents:
    - $-(2^4) = -(2 \times 2 \times 2 \times 2)$
      $= -16$
    - $(-4)^4 = (-4) \times (-4) \times (-4) \times (-4)$
      $= 256$
    - $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3}$
      $= \frac{1}{9}$
    - $2^{-4} = \left(\frac{1}{2}\right)^4$
      $= \frac{1}{16}$
  - powers expressed in formulas:
    - when the length of one side of the base of a cube = 12.5 cm, then
      volume of a cube = (*side length*)^3
      = (12.5 cm)^3
      = 12.5 cm × 12.5 cm × 12.5 cm
      = 1953.1 cm^3

Instructional Tips

Teachers can:

- use a variety of strategies, including representing using visual models, patterning, and coding, to build students' understanding of the relationship between the sign and size of an exponent and the value of a power;
- begin the learning with patterns involving bases that are natural numbers, then extend the learning to bases that are integers, unit fractions, and proper and improper fractions;
- extend a pattern from numeric bases to variable bases to build students' understanding of how to generalize the relationship between the sign and size of an exponent and the value of a power;
- create opportunities to illustrate why scientific notation can be useful for representing extremely large and extremely small numbers, including negative numbers; for example, instead of writing 0.000 000 002 5 m, it is simpler and more efficient to write 2.5×10^{-9} m.

Teacher Prompts

- What happens to the value of a power when the size of the exponent decreases by 1?
- What happens to the exponent of the power when the value of the power is multiplied by the base of the power? (For example, what happens to the exponent of 7^4 when you multiply this power by 7?)
- What is the value of a power with an exponent of 0? How do you know?
- How would you show in a different way that $3^{-2} = \frac{1}{9}$?
- What are some situations where you have noticed numbers written in scientific notation?
- What do you notice when you divide or multiply a number repeatedly by 10? What is the connection to showing this in scientific notation?
- How would you use patterning to write 23.7×10^5 in scientific notation?

Sample Tasks

1. Give students different base numbers. Have each student make their own patterning chart like those in the example above.

2. Have students use powers to form as many expressions equal to 1000 as they can.

3. Have students determine, using a strategy of their choice, which number is smaller, -4×10^3 or 4×10^{-3}.

4. Have students explain how they would write 0.005 689 and 479 000 in scientific notation.

B2.2 Powers

analyse, through the use of patterning, the relationships between the exponents of powers and the operations with powers, and use these relationships to simplify numeric and algebraic expressions

Teacher supports

Examples

- **simplifying operations with powers by examining patterns in the expanded forms of powers**:
  - product of powers (addition of exponents):
    - $3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^{(5 + 2)} = 3^7$
    - $3^5 \times 3^{-2} = (3 \times 3 \times 3 \times 3 \times 3) \times \frac{1}{3 \times 3} = 3^{5 + (-2)} = 3^3$
    - $a^x \times a^y = a^{x + y}$
  - quotient of powers (subtraction of exponents):
    - $\frac{4^5}{4^3} = \frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4}$
      $= 4 \times 4$
      $= 4^2$
  - power of a power (multiplication of exponents):
    - $(5^2)^3 = 5^2 \times 5^2 \times 5^2 = (5 \times 5) \times (5 \times 5) \times (5 \times 5) = 5^6$
    - $(a^x)^y = a^{(x \times y)}$
- **numeric expressions**:
  - expressions composed of rational bases and integer exponents; for example, $2^{-4} \times 2^5$, $10^7 \div 10^5$, $((\frac{1}{3})^2)^4$, $\frac{(-2.3)^5 \times (-2.3)^{-2}}{(-2.3)^3}$
- **algebraic expressions**:
  - expressions involving integer exponents; for example, $x^5 \times x^{-2}$, $(y^{-2})^2$, $(xy^2)^2$, $\frac{x^{-3} \times x^4}{x^2}$

Instructional Tips

Teachers can:

- emphasize the reasoning behind the relationships between multiplication, division, and power of a power involving exponents;
- use numeric expressions based on real-life examples to highlight bases that are commonly used (e.g., base 10 in scientific notation);
- organize the learning in stages to develop students' understanding of the relationships (e.g., begin the learning with exponents that are positive integers and then move to negative integers; begin with simplifying numeric expressions and then move to algebraic expressions);
- consider connecting the learning in this expectation to future learning about domain; for example,

$m^5 \times m^{-2}$ does not allow $m = 0$, since division by 0 is undefined, but simplifying to m^3 seems to make $m = 0$ allowable.

Teacher Prompts

- How does using the expanded forms of powers help you to understand the rules for operations with powers?
- How are 4^{500} and 2^{1000} the same? How are they different? Can you identify other pairs of powers that exhibit the same characteristics?
- What two powers could you multiply together to get 9^{-4}? Is there another set of two powers that you could use?
- How would you use a patterning approach to simplify 5^9 divided by 5^3?
- What strategies would you use to determine a possible way to complete the mathematical statement $\frac{(x^{\square})^{\square}}{x^{\square}} = x$?

Sample Tasks

1. Have students use a patterning approach to show that $3^2 \times 3^4 = 3^6$.

2. Have students use a patterning approach to justify that $(-0.5)^0 = 1$.

3. Give students the answer 4^9, and have them come up with potential questions.

4. Have students fill in the blanks to make the statements true:

- $a^5 \times \square^{\square} \times \square^{\square} = \square^2$
- $\square^{\square} \div \square^3 = b^{\square}$
- $(\square^{\square})^2 = c^{\square}$

Overall expectation

B3. Number Sense and Operations

apply an understanding of rational numbers, ratios, rates, percentages, and proportions, in various mathematical contexts, and to solve problems

Specific expectations

By the end of this course, students will:

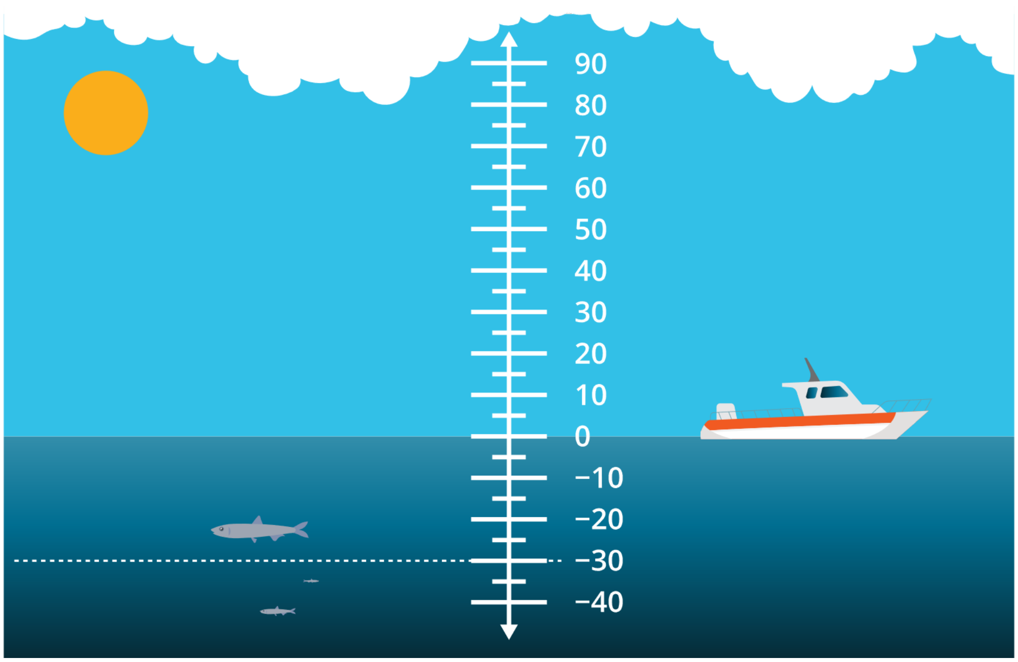
B3.1 Rational Numbers

apply an understanding of integers to describe location, direction, amount, and changes in any of these, in various contexts

Teacher supports

Examples

- **various contexts:**
  - travel in a line (change in direction and location)
  - difference in temperature (change in amount)
  - monetary transaction (change in amount)
  - gain or loss of points (change in amount)
  - direction of a rotation (clockwise or counterclockwise)
  - change in sea level (change in direction and location):



Instructional Tips

Teachers can:

- highlight the concept that any number, including an integer, can be applied to many different contexts, and that negative-number contexts can be thought of as the opposite of positive-number contexts; for example, +5 can represent how much money you have saved or added to your account, while −5 tells how much you have spent or taken out of your account;
- support students in incorporating visual representations and manipulatives to illustrate and determine location, direction, amount, and changes in these;
- emphasize the meanings of both the sign and size of an integer being used to describe location, direction, and amount when performing operations with integers.

Teacher Prompts

- Why is it sometimes helpful to use negative numbers to describe real-life situations? What does the negative sign represent in all of those situations?
- What is the purpose of benchmarks such as zero? One?
- How would you represent –50?
- How are 50 and –50 the same? different?
- If the answer is –7, what might the question be?
- How could you use a negative integer to describe an object that is 30 metres below the surface of a lake? Or to state that a company is in debt by $10 000?

Sample Tasks

1. Have students work in groups to describe scenarios in which negative integers might be used.

2. Have students describe the changes in the direction, location, and distance travelled by a train starting from 3 km west of a station and travelling to 5 km east of the station.

3. Have students represent the following:

   - paying $50 from your bank account that had $400
   - a temperature drop of 5 °C from 25 °C
   - being paid $120 after owing $30

4. Present the following scenario: At 5:00 a.m., the temperature was −3 °C.

   - Have students use a number line to determine the change in the temperature from 5:00 a.m. to 2:00 p.m. if the temperature was 6 °C at 2:00 p.m.
   - Have students use counters or tiles to determine the temperature at 10:00 a.m. if the temperature rises steadily by 2 °C every hour.

5. Present the following scenario: A chequing account has a balance of $250 at the start of the month. Weekly withdrawals of $75 are made for four weeks. Have students determine the balance in the account at the end of the four weeks.

B3.2 Rational Numbers

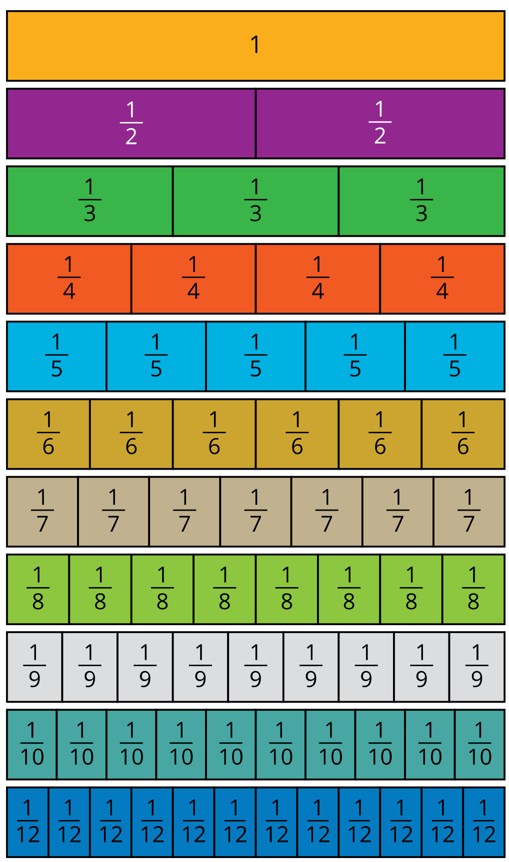
apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools

Teacher supports

Examples

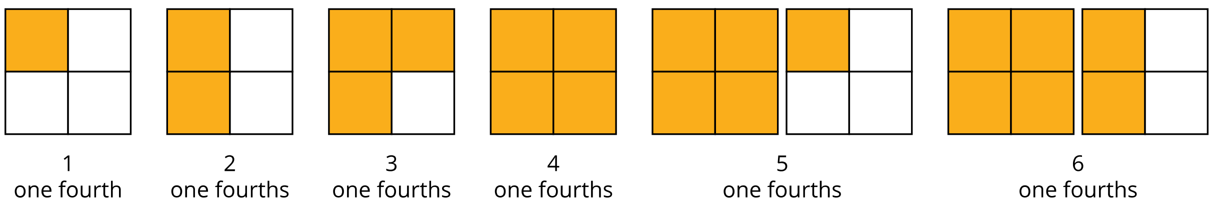
- **relationship of unit fractions to other fractional amounts:**
  - $\frac{3}{4} = \frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3\left(\frac{1}{4}\right)$ = 3 one fourths
  - $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 + \frac{2}{3} = 1\frac{2}{3} = 5\left(\frac{1}{3}\right)$ = 5 one thirds

- **tools:**
  - fraction strips:



- number line:

- area model:



- measuring tools:
  - tape measures
  - calipers
  - measuring cups and spoons
  - scoops

Instructional Tips

Teachers can:

- provide students with opportunities to consolidate their earlier learning about fractions in a wide variety of contexts, including by making connections to other strands;
- highlight the different meanings of fractions: part-whole relationships, part-part relationships, quotients, and operators;
- support students in developing their fractional thinking across all strands, such as applying an understanding of fractions to express the slope of a line;
- highlight the utility of unit fractions by incorporating purposeful questioning in their work with students;
- support students in making comparisons between unit fractions and unit rates.

Teacher Prompts

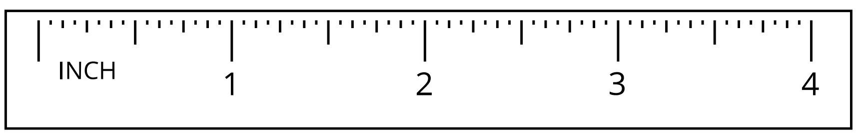
- What would you do if you only had a $\frac{1}{4}$ cup measuring cup when you were following a recipe that called for $\frac{3}{4}$ cup of flour? or $1\frac{1}{2}$ cups of flour?
- How do you measure $\frac{1}{3}$ of a cup on a $\frac{1}{2}$ cup stick of butter?

- What strategies would you use to identify the location of seven fourths ($\frac{7}{4}$) on an imperial tape measure?
- How do you know two fractions are equivalent?
- Which two fractions would you add to get an answer of $\frac{11}{12}$?

Sample Tasks

1. Have students identify the following fractional amounts on a tape measure:

- $\frac{1}{8}$ inch
- $\frac{3}{16}$ inch
- $\frac{3}{4}$ inch
- $\frac{5}{4}$ inches
- $2\frac{1}{8}$ inches
- $\frac{7}{2}$ inches



2. Have students come up with fractions that can be multiplied together to get a result of $\frac{3}{8}$. Have them come up with two numbers that can be multiplied together to get a result of $\frac{3}{8}$, with one number being greater than 3.

3. Present the following scenario: A bowl contains one scoop of mango ice cream and two scoops of green-tea ice cream. Have students describe the part-part and part-whole relationships they notice.

4. Have students determine the amount of time it would take to run 5 kilometres if they could run $\frac{1}{2}$ kilometre in 2 minutes.

5. Have students determine how many cups of sugar they would need to make a recipe that calls for 3 cups of flour, if they need to add $\frac{1}{4}$ cup of sugar for every $\frac{1}{2}$ cup of flour.

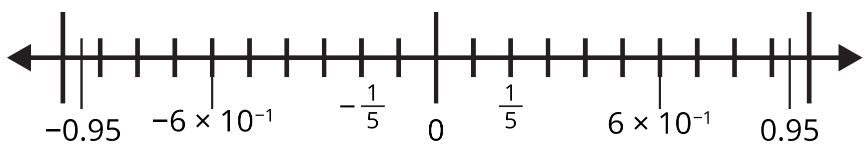
B3.3 Rational Numbers

apply an understanding of integers to explain the effects that positive and negative signs have on the values of ratios, rates, fractions, and decimals, in various contexts

Teacher supports

Examples

- **various contexts for positive and negative signs:**
  - lines with slopes that are positive and negative integers, fractions, or decimal values
  - a monetary transaction that involves spending more than the balance in the account
  - rates with negative signs; for example, an object with a velocity of 60 km/h is moving at this rate in the positive direction, as defined by the coordinate system, while a velocity of –60 km/h means the object is moving at this rate in the opposite direction
  - plus–minus, a ratio used to measure a sports player's impact on a game, represented by the difference between their team's total scoring versus their opponent's, when the player is in the game
  - comparing values on a number line to the left and right of zero:



Instructional Tips

Teachers can:

- draw students' attention to the meaning of the negative sign on various numbers in different contexts;
- illustrate different ways of representing the same negative rational number (e.g., $\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$);
- support students in representing, understanding, and performing operations with negative fractions, decimals, and integers.

Teacher Prompts

- What situations could be described by negative ratios, rates, fractions, and decimals in real life?
- What does a 25% discount or markup mean?
- In what other ways can you represent the number –1.003?
- How could you apply what you know about integer operations to predict the signs of the following equations?

  - $-\frac{1}{2} \times \frac{1}{4}$
  - $-\frac{1}{2} \times \left(-\frac{1}{4}\right)$
  - $-\frac{1}{2} + \frac{1}{4}$
  - $-\frac{1}{2} - \frac{1}{4}$
- What does the negative sign represent when we consider an object falling at –4.9 m/s^2?
- How would you explain the relationship between $\frac{-3}{5}$, $\frac{3}{-5}$ and $-\frac{3}{5}$?
- Where would you locate –7.36 on a number line? How about $-2\frac{7}{8}$? What strategies did you use? How would your process be the same if you were locating 7.36 and $2\frac{7}{8}$, and how would it be different?

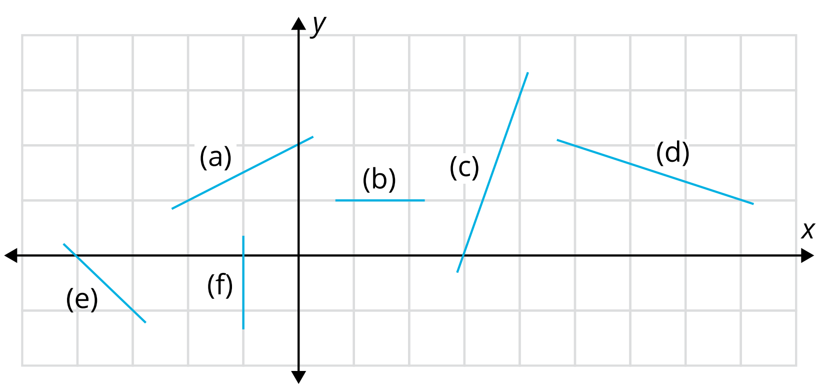
Sample Tasks

1. Present the following scenario: A train approaching a station travels at 100 km/h and slows down at a constant rate of change of −10 km/h. Have students determine the distance it would have to begin its deceleration.

2. Have students create a mathematical statement that describes the changes in depth of a submarine that descends 118.4 m, then goes down another 54.2 m, and then comes up 68.3 m.

3. Have students describe sports situations where positive and negative ratios, rates, fractions, or decimals are used and identify what is being compared.

4. Have students determine the slope of each line segment in the image below and describe the steepness and direction of each line segment.



B3.4 Applications

solve problems involving operations with positive and negative fractions and mixed numbers, including problems involving formulas, measurements, and linear relations, using technology when appropriate

Teacher supports

Examples

- **operations with positive and negative fractions and mixed numbers:**
  - addition and subtraction with:
    - like denominators
    - unlike denominators with a common divisor
    - unlike denominators
  - multiplication:
    - where the numerator of one term is the denominator of the other
    - with any denominator
  - division:
    - by a unit fraction with a like denominator
    - by a denominator with a common divisor
    - by any denominator
  - powers of fractions
    - with whole number exponents

Instructional Tips

Teachers can:

- emphasize that the same reasoning that is applied to deciding whether the sum, difference, product, or quotient of a pair of integers is positive or negative can also be applied to all operations on rational numbers;
- pose problems from various contexts to support students in developing number sense;
- encourage students to incorporate various representations in their reasoning about problems and operations;
- create opportunities for students to form mental representations and use mental mathematics in order to develop procedural fluency;
- support students in making sense of the reasoning behind procedures involving operations with fractions;
- support students in developing their estimation skills and recognizing benchmark numbers through performing collaborative learning tasks.

Teacher Prompts

- When creating a table of values for the linear relation $y = -\frac{3}{4}x + \frac{1}{4}$, what *x*-values would be considered "easiest" to use? How do you know?
- What steps would you follow to get the output of the relation $y = \left(-\frac{1}{4}\right)x + \frac{3}{2}$ for various inputs using technology?
- What strategies would you use to determine $\frac{1}{2}$ of −14 and $\frac{3}{7}$ of 49?
- If you are 5′ 3″ and you are growing or shrinking $\frac{1}{8}$ of an inch each year, how would you calculate your new height?
- What steps could you follow when subtracting a mixed number from another mixed number with a different denominator?
- What are some values that would make the following statement true? ____ is $\frac{2}{7}$ of _____.

Sample Tasks

1. Present the following scenario: A recipe calls for $2\frac{1}{2}$ cups of flour, $1\frac{1}{4}$ cups of granulated sugar, and $\frac{2}{3}$ cup of butter. Have students determine the amount of ingredients in the mixing bowl using relational rods, fraction bars, and other tools. Have students repeat the task, changing the ingredient measurements to grams and millilitres.

2. Present the following scenario: A school play requires 5 costumes, each requiring $1\frac{3}{4}$ yards of fabric. Have students use various tools to determine how much more fabric will be needed if the play receives a donation of 8 yards of fabric.

3. Have students determine the perimeter and the area of a rectangular piece of plywood with a length of $3\frac{3}{4}$ ft and a width of $1\frac{1}{2}$ ft.

4. Have students create a table of values for the linear relation $y = -\frac{3}{4}x + \frac{1}{4}$ for *x* = −2, −1, 0, 1, 2, 3, 4.

5. Present the following scenario: A 70 L rain barrel loses $\frac{3}{4}$ L of water every minute. Have students create a table of values, using technology, that shows the amount of water in the barrel from the time the barrel is full to when it is empty.

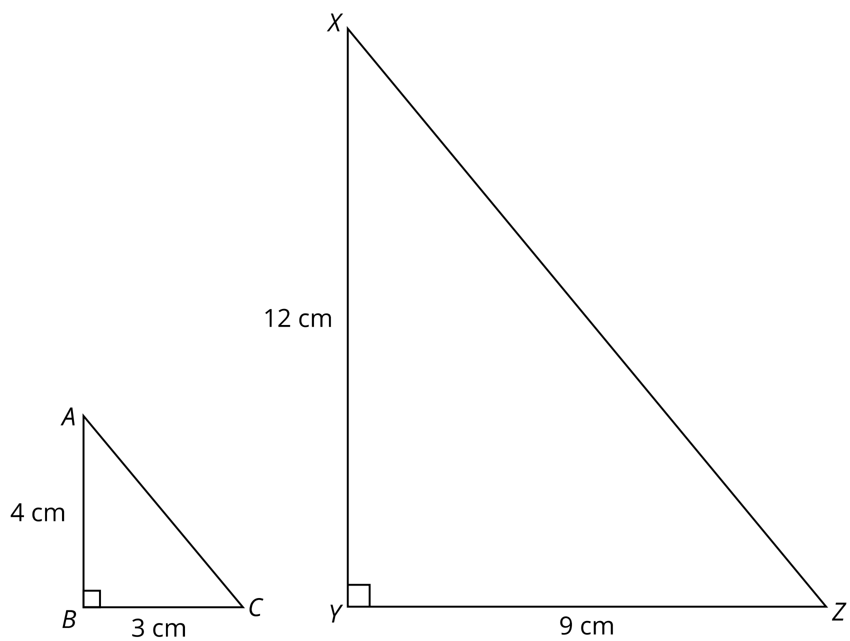
B3.5 Applications

pose and solve problems involving rates, percentages, and proportions in various contexts, including contexts connected to real-life applications of data, measurement, geometry, linear relations, and financial literacy

Teacher supports

Examples

- **various contexts:**
  - determining missing lengths in similar triangles:



  - exploring the golden ratio in nature
  - flow rates:
    - e.g., determining when the water level of a river will drop to 0.28 m from 1.08 m if it is receding at a rate of 0.05 m/day
  - probability:
    - e.g., calculating the probability of two independent events: rolling a 4 on a die and getting tails when flipping a coin
  - measurement:
    - e.g., comparing the changes in the surface area and volume of a box if one dimension (e.g., the height) increases or decreases
  - making purchases:
    - e.g., prices involving a discount and sales tax
    - e.g., comparing costs by calculating unit rates

- comparing fuel efficiency or energy consumption:
  - e.g., different vehicles, in different driving conditions

Instructional Tips

Teachers can:

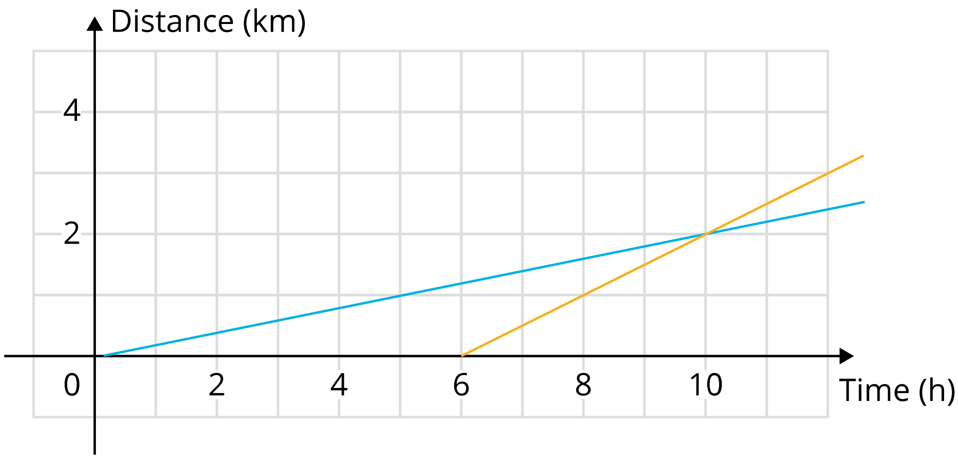
- highlight the concept that rates and ratios, including percentages, describe comparisons;
- emphasize the cross-strand and cross-curricular connections that illustrate real-life applications of number concepts (e.g., the use of proportions between physical quantities in science, such as describing density as the ratio of mass to volume; the use of rates in data analysis and linear relations; and the use of rates and percentages in financial literacy);
- support students in selecting an appropriate tool or strategy for the problem they want to solve, such as using grid paper or ratio tables.

Teacher Prompts

- How do you know whether a given situation involves rates, percentages, or proportions?
- Describe two different types of situations where knowing the unit rate would be beneficial.
- Is the ratio between the perimeter of a square and its diagonal the same for any two squares? How do you know?
- Given a cylindrical container that holds $\frac{1}{2}$ L and has a radius of 5 cm, determine the height of the original container and then find the height of containers that hold double the original volume and 25% more than the original volume.
- Which slope is steeper: 12% or $\frac{8}{50}$? How do you know?
- How do you know how these two mathematical statements compare, without calculating the answers: 75% of 28 or 28% of 75?
- What are some values that would make the following statement true? $32 is ____% of ______.
- What strategies would you use to determine which of the below is the better deal?

Sample Tasks

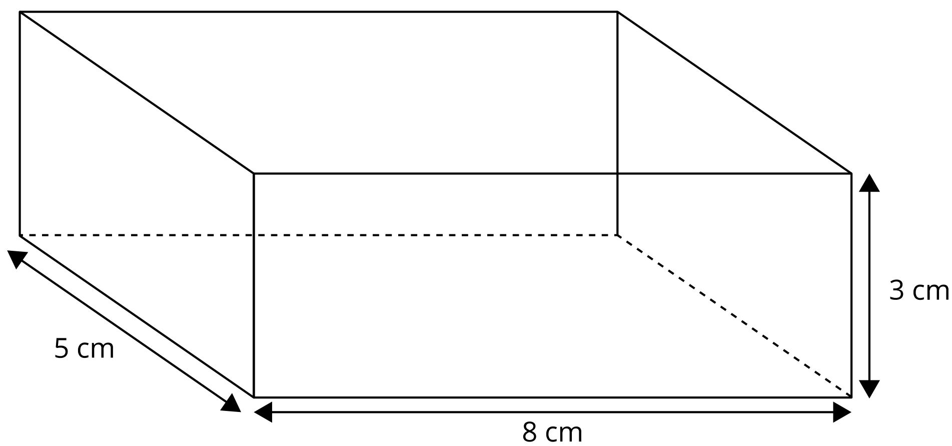
1. Have students compare the slopes of two different lines and discuss the differences using words, numbers, and equations.



2. Have students identify the best deal in the images below using a variety of strategies.

3. Have students discuss when they would rather have 20% off the price of an item or a $20 discount on an item.

4. Have students compare the percentage change in the surface area and the volume when the height of the box in the below image doubles.



5. A one-trip bus fare is $2.55 and a weekly pass for students costs $27.50. Have students determine which payment method is preferred in each of the following scenarios:

   - taking the bus to and from school for 5 days
   - taking the bus to school for 5 days, going to work after school on the bus for 3 of those days, and coming home on the bus on each of the 5 days.

C. Algebra

Overall Expectations

By the end of this course, students will:

C1. Algebraic Expressions and Equations: demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

C2. Coding: apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands

C3. Application of Relations: represent and compare linear and non-linear relations that model real-life situations, and use these representations to make predictions

C4. Characteristics of Relations: demonstrate an understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate

Overall expectation

C1. Algebraic Expressions and Equations

demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

Specific expectations

By the end of this course, students will:

C1.1 Development and Use of Algebra

research an algebraic concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context

Teacher supports

Examples

- **stories that students may share involving algebraic concepts:**
  - use of variables:
    - Today, symbols like *x* and *y* are used to represent unknown quantities (variables). In a similar way, a scroll of hieroglyphics from ancient Egypt (1650 BCE) shows the use of a symbol to represent an unknown value called a “heap” or a “quantity”.
  - patterns and binary code:

        - Algebra and binary code are closely connected to weaving, which exists in most cultures and communities. Joseph Marie Jacquard (1752–1834) invented the Jacquard loom, often referred to as a precursor to the computer because it wove patterns based on instructions encoded on a series of punch cards. The activity of weaving itself involves mathematical thinking. A weaver uses algebraic thinking, often tacitly, to decide which threads to pick up based on what is needed for the intended pattern, and the act of picking up a thread or leaving it is a binary decision.
    - generalizing relationships and rates of growth:
        - Knowing the relationship between the girth of a tree and its height can be helpful in determining the height of a tree without cutting it down. Determining this relationship requires observing the trees that grow locally in order to understand the growth rates of different species and how different growing conditions might affect these growth rates. This knowledge is used in various cultures by people who harvest wood for cultural, creative, and practical purposes so that they are able to harvest the right wood for the intended use and so that it can be done in a way that is sustainable.

Instructional Tips

Teachers can:

- on an ongoing basis, both formally and informally, encourage students to bring into the classroom the real-life stories and experiences they have gathered about the mathematical concepts they are learning, in order to enhance their understanding of these concepts, make connections between them, and connect them to real life;
- build an authentic and inclusive learning environment where students are encouraged to learn about knowledge systems from around the world, including Indigenous ways of knowing.

Note

Students can seek out stories and real-life experiences through conversations with people in their families or communities, or through print and digital resources. They may need guidance on seeking information about new perspectives on mathematics. The aspect of student choice in this expectation may also involve the teacher taking the stance of a co-learner as they support students in exploring stories of various cultures.

Teacher Prompts

- How is this concept relevant to you?
- What did you find most interesting about your story?
- If you faced challenges while researching this concept, what were they?

- Describe some ways you can connect the concept you researched to your own learning, careers that you know about, the natural world, or your daily life.
- What do you think connects the different stories shared in class and the mathematical concepts you have encountered?
- What are some similarities and differences between your story and concepts researched by your peers?

Sample Tasks

1. Have students brainstorm possible algebraic concepts that they could research. Ask them to choose a concept of interest from the list and gather information about its socio-historic development in a culture of their choice. Once students have gathered their information, have them decide how they would like to tell the story of the development of the concept to the class.

2. Have students collaborate to create a collage of images showing the relevance in current contexts of the various algebraic concepts they researched. Each image should be connected to a specific algebraic concept, with a brief explanation of the connection. Students might make connections to the arts, architecture, engineering, science, business, the natural world, and so on.

C1.2 Algebraic Expressions and Equations

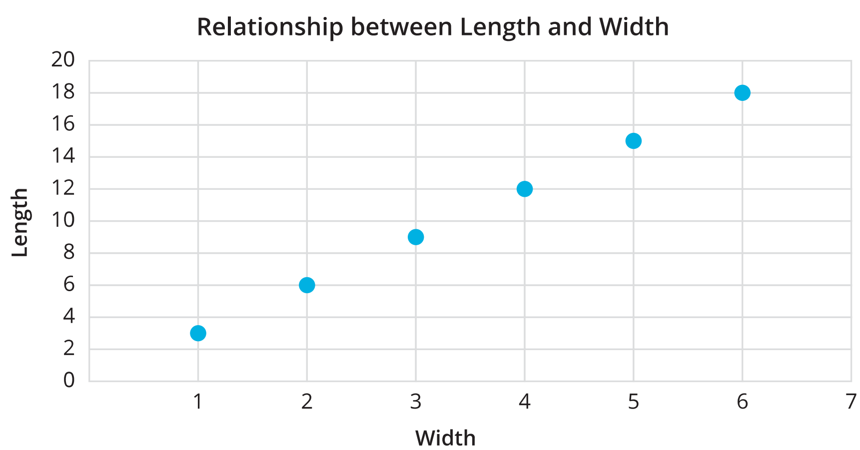
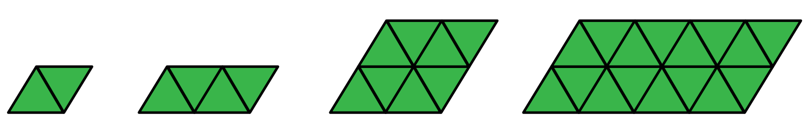
create algebraic expressions to generalize relationships expressed in words, numbers, and visual representations, in various contexts

Teacher supports

Examples

relationships expressed in words	algebraic expressions that generalize these relationships
The length of a rectangle is three times its width.	length = 3 × width or $l = 3w$ or $$\text{width} = \frac{\text{length}}{3}$$
three less than double the sum of two numbers	$2(x + y) - 3$

<table>
<tr><th>relationships expressed in numbers</th><th>algebraic expressions that generalize these relationships</th></tr>
<tr><td>a table of values:
<table><tr><th>Width</th><th>Length</th></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>9</td></tr></table></td><td>length = 3 × width
or
$l = 3w$</td></tr>
<tr><td>a sequence:
1, 4, 9, 16, 25, 36</td><td>1, 4, 9, 16, 25, 36, …, n^2
or
term value = (term number)^2
or
$t = n^2$</td></tr>
</table>

<table>
<tr><th>relationships represented visually</th><th>algebraic expressions that generalize these relationships</th></tr>
<tr><td>
</td><td>length = 3 × width
or
$l = 3w$</td></tr>
<tr><td>number of tiles in term n:

1 2 3 4</td><td>2^n
or
$T = 2^n$</td></tr>
</table>

Instructional Tips

Teachers can:

- provide concrete representations and contexts to support students when they are generalizing relationships, especially non-linear relationships;
- share examples of how expressions are used in coding and how, depending on the coding language, they can be represented using words, abbreviated text, or symbols;
- provide examples of relationships that students might represent in different ways, such as generalizing an expression for perimeter or a given visual representation, to support the concept of equivalent algebraic expressions (see C1.3);
- support students in making connections to Strand B: Number by providing patterns and number relationships for them to generalize;
- provide opportunities for students to:
    - use concrete materials such as algebra tiles, pattern blocks, colour tiles, or beads to represent and generalize relationships;
    - generalize algebraic expressions in various ways, such as by using words, abbreviations, and/or symbols.

Teacher Prompts

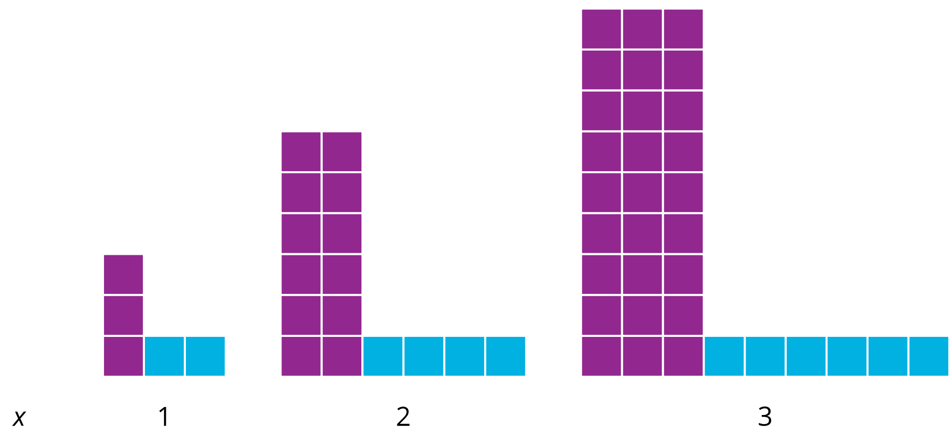
- How can generalizing a mathematical relationship be helpful?
- How can describing a mathematical relationship in words help you generalize it with symbols?
- What are some of your observations about this relationship? How can those observations help you generalize the relationship?
- Write a different algebraic expression to represent the same relationship.
- What algebraic expression can you create to represent elements of this set of numbers (e.g., even numbers, perfect squares, triangular numbers)?

Sample Tasks

1. Provide students with a set of cards with various relationships expressed in words, with numbers, and with visual representations, as well as cards with corresponding algebraic expressions. Have them match the relationship cards to the corresponding expression cards. Consider leaving out some information on the cards so that students can complete them to show understanding of the relationship.

2. Provide students with a visual representation and several possible algebraic expressions. Have them match the appropriate expression(s) to the visual representation and justify their selections. For example:

- The first three terms of a pattern are shown below, where x represents the term number. Which of the following expressions represents this pattern? Justify your answer. (Note that both b) and d) are correct in this example, but students may justify that, for example, b) connects to the visual representation better than d), depending on how they interpret it).
  a) $3x + 2x$
  b) $3x^2 + 2x$
  c) $3x(x + 2)$
  d) $x(3x + 2)$



3. Pose a number riddle to students, like the one described below, and have them test it with a few different numbers. Then have them create algebraic expressions with algebra tiles and with symbolic notation for each stage of the riddle. Ask them to explain how generalizing the expressions helps them understand how the riddle works.

   Example of a number riddle:

   - Pick a number.
   - Add 5 to your number.
   - Double your result.
   - Subtract 2.
   - Divide your answer by 2.
   - Subtract your original number.
   - What is your answer?

4. Have students represent an algebraic expression of their choice using algebra tiles, and then have them ask a partner to write an expression for their representation.

5. Pose a problem for which generalizing with an algebraic expression might be helpful. For example: At the beginning of a gathering, each person in the room greets every other person exactly once. How many greetings are there if there are 5 people in the room? 10 people in the room? What

expression could you use to determine the number of greetings for any number of people in the room?

C1.3 Algebraic Expressions and Equations

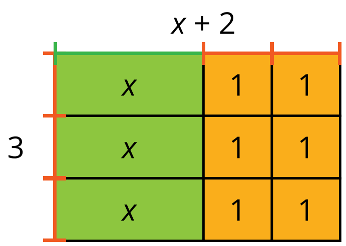
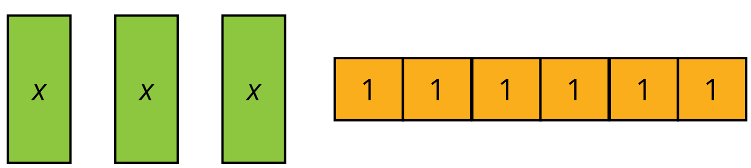
compare algebraic expressions using concrete, numerical, graphical, and algebraic methods to identify those that are equivalent, and justify their choices

Teacher supports

Examples

- **algebraic expressions:**
  - non-simplified polynomials with a single variable involving addition or subtraction; for example, compare $3x + 5x$ with $-x + 6x$
  - simplified polynomials with a single variable and non-simplified polynomials involving multiplication; for example, compare $2m + 10$ with $2(m + 5)$
  - polynomial expressions involving multiple operations; for example, compare $-2x(x + 5) - 2$ with $-2(x^2 + 5x - 1)$
  - algebraic expressions involving integer exponents; for example, compare $\frac{(a^6)(a^3)}{a^2}$ with $(a^3)^4(a^{-5})$

- **comparing using concrete methods:**
  - represent and compare algebraic expressions using concrete tools such as algebra tiles, colour tiles, and pattern blocks:
    - compare $3(x + 2)$ and $3x + 6$ using algebra tiles:

<table>
<tr><th>3(x + 2)</th><th>3x + 6</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

These expressions are equivalent because they are represented by the same number of each tile.

- **comparing using numerical methods:**
  - use substitution to test the output for various inputs:
    - compare $2(x - 3) + 3x$ and $5(x - 2) + 2$ using substitution:

testing with $x = 3$	$2(x - 3) + 3x$ $= 2(3 - 3) + 3(3)$ $= 0 + 9$ $= 9$	$5(x - 2) + 2$ $= 5(3 - 2) + 2$ $= 5 + 2$ $= 7$

These expressions are not equivalent because the same input gives different outputs.

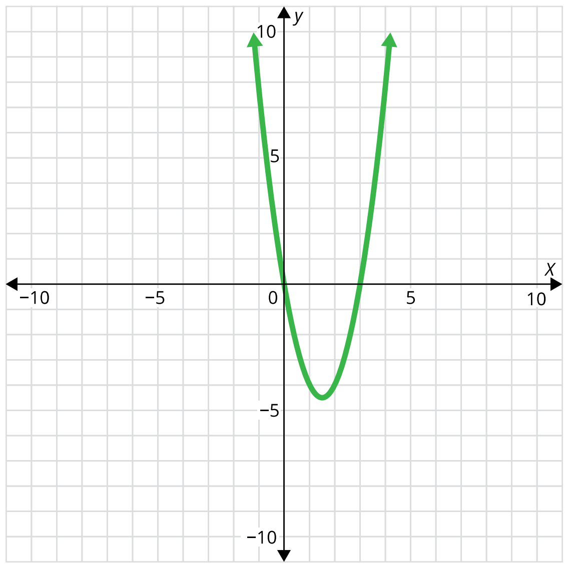
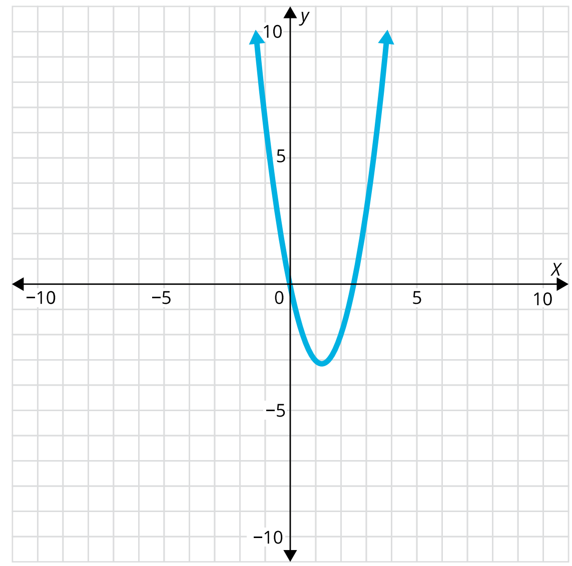
- use a table of values to test the output for various inputs:
  - compare $x(x - 4)$ and $x^2 - 4x$ using a table of values:

x	$x(x - 4)$	$x^2 - 4x$
0	$0(0 - 4)$ $= 0$	$0^2 - 4(0)$ $= 0$
1	$1(1 - 4)$ $= 1(-3)$ $= -3$	$1^2 - 4(1)$ $= 1 - 4$ $= -3$
5	$5(5 - 4)$ $= 5(1)$ $= 5$	$5^2 - 4(5)$ $= 25 - 20$ $= 5$
10	$10(10 - 4)$ $= 10(6)$ $= 60$	$10^2 - 4(10)$ $= 100 - 40$ $= 60$
$\frac{1}{2}$	$\frac{1}{2}\left(\frac{1}{2} - 4\right)$ $= \frac{1}{2}\left(-\frac{7}{2}\right)$ $= -\frac{7}{4}$	$\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right)$ $= \frac{1}{4} - 2$ $= -\frac{7}{4}$
-2	$-2(-2 - 4)$ $= -2(-6)$ $= 12$	$(-2)^2 - 4(-2)$ $= 4 + 8$ $= 12$
-3	$-3(-3 - 4)$ $= -3(-7)$ $= 21$	$(-3)^2 - 4(-3)$ $= 9 + 12$ $= 21$

Although these expressions are equivalent for the tested inputs, it is impossible to determine for certain whether they are always equivalent without using another method.

- **comparing using graphical methods:**
  - graph the expressions to determine whether the graphs are identical:

- compare $2x(x-3)$ and $2x^2-5x$ by graphing:

$y = 2x(x-3)$	$y = 2x^2 - 5x$
	

These two expressions are not equivalent because the graphs that represent them are different.

- **comparing using algebraic methods:**
  - simplify the expressions:
    - compare $(a^5)(a^3)$ to $(a^2)^4$ by expanding and simplifying:

$(a^5)(a^3) = (a \cdot a \cdot a \cdot a \cdot a)(a \cdot a \cdot a)$ $= a^8$	$(a^2)^4 = (a^2)(a^2)(a^2)(a^2)$ $= (a \cdot a)(a \cdot a)(a \cdot a)(a \cdot a)$ $= a^8$

- compare $(a^5)(a^3)$ to $(a^2)^4$ by applying exponent laws:

$(a^5)(a^3) = a^{5+3}$ $= a^8$	$(a^2)^4 = a^{2 \times 4}$ $= a^8$

These expressions are equivalent because they can be simplified to the same term.

Instructional Tips

Teachers can:

- provide students with scenarios, patterns, and visuals that can be generalized in a variety of ways in order to generate algebraic expressions for comparison;
- facilitate discussions about the similarities and differences between expressions when comparing them;
- encourage students to use various methods to compare expressions, and facilitate a discussion about the possible strengths and limitations of each method; for example, whether testing the expressions with a small number of values is enough to determine equivalence;
- use familiar measurement formulas to support students in making connections to equivalent algebraic expressions; for example, the perimeter of a rectangle can be represented as $P = l + w + l + w$ or $P = 2l + 2w$ or $P = 2(l + w)$;
- introduce scenarios that could expose common errors; for example, compare $2(x – 3)$ and $2x – 3$;
- support students in developing an understanding of how different expressions can represent the same relationships and how, at times, some representations are more useful than others;
- provide opportunities for students to:
  - listen to and reflect on their peers' reasoning, representations, and strategies;
  - use coding, digital tools, and various concrete materials (e.g., algebra tiles, colour tiles, interlocking cubes) to compare algebraic expressions.

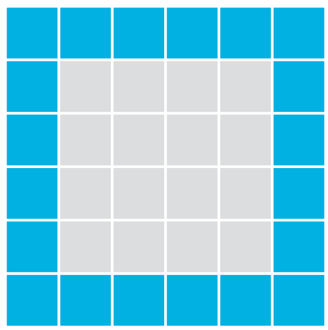
Teacher Prompts

- Represent the expressions $x^2 + x^2 + 4x$ and $2x(x + 2)$ with concrete materials. What do you notice about the representations? What is the same and what is different?
- What do you notice when you substitute the same value into the expressions $8x^3$ and $(2x)^3$ and evaluate them?
- Is it possible to prove that expressions are equivalent by substituting the same value into both and checking whether they result in the same output? Why or why not?
- Examine two expressions and predict whether they are equivalent. What did you consider when making your prediction?
- What are some reasons that it might be helpful to write an expression in a different but equivalent way?
- What do you notice when you compare two expressions graphically? How can you use graphs to determine whether algebraic expressions are equivalent?
- Which strategy do you feel most confident using to compare expressions? Why?
- One pattern is represented by the expression $2x + 7$, and another pattern is represented by the expression $x^2 + 7$. How are these two expressions similar, and how are they different? How are the two patterns similar and different?

Sample Tasks

1. Provide students with a set of algebraic expressions on cards. Have them compare the expressions using a method of their choice and then group cards with equivalent expressions. Ask them to create additional expressions for each group, explaining why each expression belongs in that group.

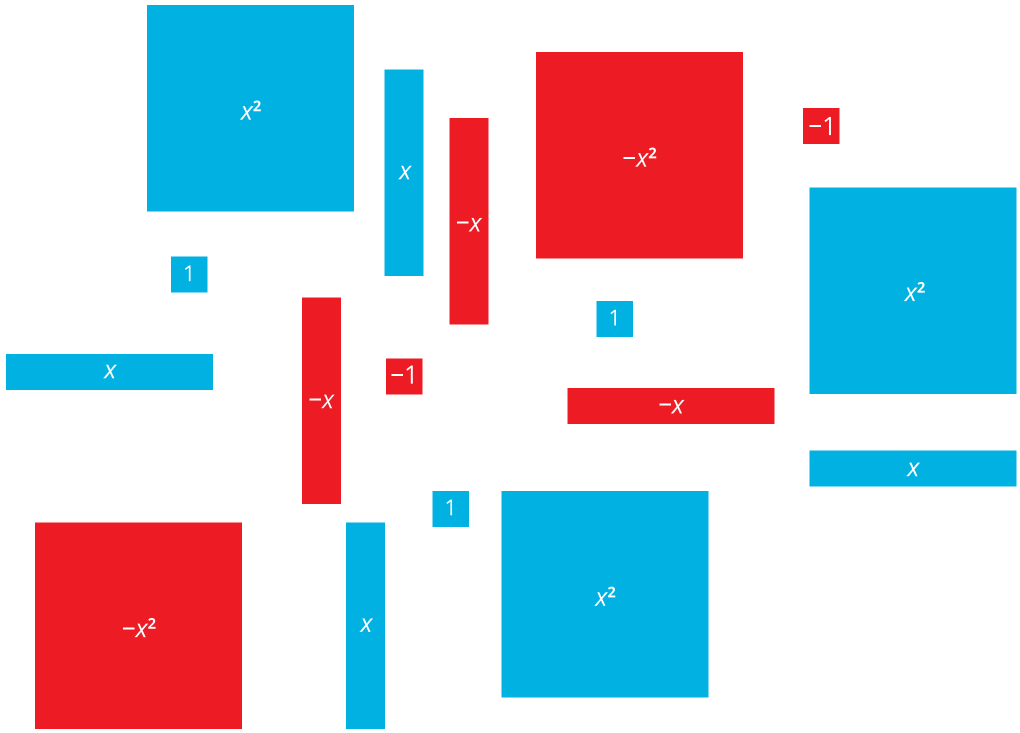
2. Have students generate different algebraic expressions for the same visual pattern and then compare the expressions to see if they are equivalent. Start with a visual representation that allows students to notice the relationship in multiple ways. For example: Generate an expression to determine how many tiles are in the border of an *n* by *n* version of the layout below. How does your expression connect to the way you understand the pattern? How are the various expressions the class generated similar, and how are they different?



3. Have students compare different versions of formulas that are used in geometry and measurement, such as formulas for the perimeter of a rectangle or for the area of a trapezoid. For example: Are the following formulas for the area of a trapezoid equivalent?

   - $A = h\left(\frac{a+b}{2}\right)$
   - $A = \frac{h}{2}(a+b)$
   - $A = \frac{1}{2}h(a+b)$

4. Provide students with a collection of algebra tiles. Ask them to write three different equivalent expressions for the collection of tiles and justify how they know that they are equivalent. For example, the collection might look like this:



C1.4 Algebraic Expressions and Equations

simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts

Teacher supports

Examples

- **simplifying algebraic expressions:**
  - add and subtract monomials of degree 1, 2, or 3; for example, $3x^2 + x - 2x^2 + 5x$, $-3ab - (-2ba)$
  - multiply a monomial by a monomial; for example, $(2x)(3x^2)$
  - multiply a monomial by a binomial; for example, $2(x + 4)$
  - subtract binomials; for example, $(2x + 4y) - (x - y)$
  - simplify combinations of the above; for example, $x(x + 2) + 3(x^2 + 2x - 5)$
- **properties of operations of numbers:**
  - commutative property of addition
  - associative property of addition
  - commutative property of multiplication

    - associative property of multiplication
    - distributive property
    - properties of exponents
- **representations:**
    - linear model
    - area model
    - volume model
- **tools:**
    - number line
    - algebra tiles
    - pattern blocks
    - colour tiles
    - straws and connectors
    - computer algebra system (CAS)
    - coding

Instructional Tips

Teachers can:

- make use of concrete materials and visual representations to support students in making connections between operations with numbers and operations with algebraic terms;
- provide students with digital tools and concrete materials that they can use to represent and simplify algebraic expressions;
- encourage students to move from concrete representations to abstract ones as well as from abstract representations to concrete ones;
- support students in making connections between relationships among exponents and powers and multiplying monomials (see B2.2);
- support students in making connections between terms of degree 1 and linear models, terms of degree 2 and area models, and terms of degree 3 and volume models;
- provide students with opportunities to use simplifying to compare algebraic expressions (see C1.3).

Teacher Prompts

- What are some reasons that simplifying an algebraic expression can be helpful?
- The commutative property of addition states that you can add numbers in any order and get the same result (e.g., $-2 + 3 = 3 + (-2)$). Does the same property hold when you add two algebraic terms, such as $-2x + 3x$? Use algebra tiles to justify your answer.
- What happens when you add $2x + 1$ to $-1 - 2x$?
- How can you use an area model to represent multiplying a binomial by a whole number?

Sample Tasks

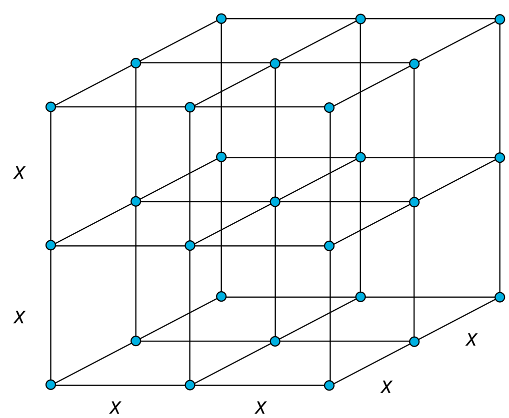
1. Provide students with a bag of various concrete materials (e.g., pattern blocks, algebra tiles). Have them write an unsimplified expression to represent the contents of the bag, and then have them simplify the expression.

2. Ask students to create the following expressions, build them with algebra tiles, and then compare their results with those of a partner:

   - two binomials that can be added to equal zero
   - two different expressions that can be simplified to $3x + 7$
   - a monomial and a binomial that can be multiplied to create a product of $6x^2 - 12x$

3. Have students construct models using straws and connectors (or similar tools) to represent the multiplication of monomials and some power rules. For example:

   - Use straws and connectors to build a model to represent $(2x)^3$, where the length of a straw represents *x,* and use the model to explain how $(2x)^3$ can be simplified to $8x^3$. The model might look like the following:



C1.5 Algebraic Expressions and Equations

create and solve equations for various contexts, and verify their solutions

Teacher supports

Examples

- **equations:**
  - linear equations; for example, $180(n - 2) = 1440$, $3x + 2 = 2(x - 5)$, $\frac{t-2}{3} = -8$

- simple equations of degree 2; for example, $A = \pi r^2$, given A
- simple equations of degree 3; for example, $V = s^3$, given V
- equations with multiple variables that require given numerical values to be substituted; for example, solve for r in $I = Prt$, given $I = 25$, $P = 500$, $t = 2$
- equations involving proportions; for example, $\frac{h}{3} = \frac{25}{12}$

- **verify solutions by:**
  - using substitution and mental math
  - using a left-side/right-side check
  - using a number line
  - using coding
  - using a graph

Instructional Tips

Teachers can:

- pose problems, and support students in posing problems, using words, numbers, and visual representations;
- introduce strategies for solving equations in context as they arise throughout the course (e.g., introduce strategies for solving a proportion when a problem involves proportions);
- provide students with digital tools and concrete materials that they can use to solve and verify equations;
- support students in using various methods and models for solving equations, such as estimation, trial and error, concrete models, a balance model, a graph, inverse operations, ratio tables, or a double open number line;
- provide opportunities for students to share strategies and listen to, make connections to, and reflect on their peers' strategies;
- share strategies for creating equivalent equations to simplify solving equations with fractions (e.g., $\frac{1}{2}x - 6 = 10$ can be rewritten as $x - 12 = 20$ by multiplying the equation by 2);
- facilitate a discussion about what degree of accuracy is necessary when solving equations, given various contexts, and which methods allow for a greater degree of accuracy;
- encourage students to reflect on the reasonableness of their answers and to choose a strategy to verify their solutions.

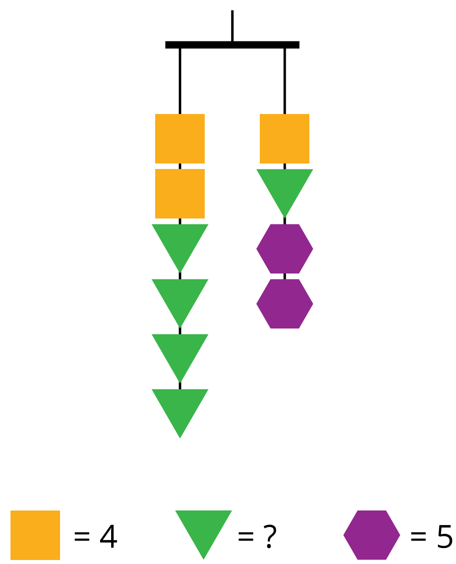
Teacher Prompts

- Identify the variables in this situation, and describe the relationship between them. What equation could you use to represent this relationship?
- What does the equal sign mean in the equation? What does it mean to solve an equation?
- What does your solution to the equation mean in this context?

- Is your answer reasonable? How do you know? How could you check whether your answer is correct?
- What strategies do you prefer to use when solving equations?
- How is the strategy you used to solve an equation similar to or different from your classmate’s?
- What is the difference between an algebraic equation and an algebraic expression?

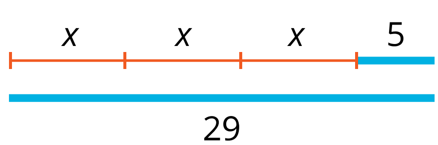
Sample Tasks

1. Have students determine the value of the triangle that would make the mobile in the image below stay balanced:



2. Then have them write an equation to represent the situation, and solve it.

3. Have students write an equation that could be represented by the visual shown below. Ask them to discuss how the visual can help them understand how to solve the equation for *x*.



4. Have students create an equation for a given solution (e.g., create an equation with a solution of $a = 5$), trade equations with a classmate to solve, and then compare strategies.

5. Throughout the course, have students create and solve equations for various contexts as they arise. Examples might include:

   - Determine the diagonal distance across a field, given the length and width.
   - Determine the amount of pay earned, given the conditions of pay.
   - Determine the fixed cost for a service, given the rate of change and one set of values.
   - Determine which term of a pattern will have 210 tiles.
   - Determine the size of an angle in a given diagram.
   - Convert from one measurement system to another.
   - Calculate the amount of simple interest earned after a period of time.
   - Solve for a missing value in a measurement formula.

6. Show students an incorrect solution for an equation, and have them identify what the error is, correct the error, and verify their solution using a strategy of their choice.

Overall expectation

C2. Coding

apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands

Specific expectations

By the end of this course, students will:

C2.1 Coding

use coding to demonstrate an understanding of algebraic concepts including variables, parameters, equations, and inequalities

Teacher supports

Examples

- **use coding to demonstrate an understanding of algebraic concepts by:**
  - executing code
  - reading code
  - altering code
  - writing code
- **use variables in code to:**
  - store data temporarily
  - access stored data
  - alter the value of stored data

- **use parameters in code to:**
  - act as placeholders to represent a quantity that influences the output of a mathematical object but is viewed as being held constant; for example, in $y = ax$, a is the parameter
  - define values that are provided by the main program to subprograms when the subprograms are called
- **use equations and inequalities in code to:**
  - assign values to memory; for example, using $y = 2x + 3$: if x is defined as 2, y is assigned the value 7
  - increment a counter; for example, using *counter* = *counter* + 1 increments the value of *counter* by 1 and updates the value of the variable
  - test equality; for example, using $y = 3x - 2$: if y is assigned the value 5 and x is assigned the value 2, this test of equality returns false because 5 is not equal to 4
  - test a range; for example, using $y < 3x - 2$: if y is assigned the value 5 and x is assigned the value 2, this test returns false because 5 is not less than 4

Instructional Tips

Teachers can:

- give students an opportunity to share their previous coding experiences and the coding tools or environments they have used;
- provide students with code that incorporates the algebraic concepts listed above and facilitate a discussion about the ways in which these concepts are used in coding;
- support student collaboration to plan their approach to writing code, using flow charts to organize their thinking;
- incorporate opportunities for students to use algebraic thinking and coding to solve various problems throughout the course;
- support students in making connections between their understanding of variables, parameters, equations, and inequalities and the use of these in coding, spreadsheets, computer algebra systems (CASs), virtual graphing and geometry tools, and text-based programming languages.

Note

The learning in C2.1, C2.2, and C2.3 is interconnected, and this should be reflected during instruction.

Teacher Prompts

- Which variables are used in this coding example?
- What values are stored in the variables, and why are they important to the program and to the problem being solved?
- What are some ways in which variables (or parameters, equations, and inequalities) are used in coding?

- Explain why understanding equations and inequalities is important when working with conditional statements in coding.
- How are equal signs used in coding?

Sample Tasks

1. Provide students with pseudocode, and ask them questions about the variables, parameters, equations, and inequalities included. For example, have them read the pseudocode below, which plots two linear relations so that students can compare them graphically, and then pose the following questions:

   - What are this code's:
     - variables?
     - parameters?
     - equations?
   - What does **totalPoints** = 10 represent?
   - What does **yValue** = **rateOfChange** * **xValue** + **initialValue** represent?

   Main program

initialValue = 3
rateOfChange = 2
totalPoints = 10
run **plotRelation** subprogram (**initialValue**, **rateOfChange**, **totalPoints**)
initialValue = 0
rateOfChange = 3
totalPoints = 10
run **plotRelation** subprogram (**initialValue**, **rateOfChange**, **totalPoints**)

plotRelation subprogram

subprogram **plotRelation** (**initialValue**, **rateOfChange**, **totalPoints**)
xValue = 0
yValue = **initialValue**
repeat **totalPoints** times
plot point (**xValue**, **yValue**)
xValue = **xValue** + 1
yValue = **rateOfChange** * **xValue** + **initialValue**

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.

2. Have students identify the variables, parameters, and/or equations needed to write code to perform a specific task. For example, ask:

    - What variables are needed to write code to represent a linear relation?
    - What parameters are needed for the line to have a positive slope?
    - What equation can be used in the code to determine the *y*-value of a linear relation?

3. Give students a short program to run. Before they look at the code, have them predict, based on the outcome from running the program, what equations, variables, parameters, and inequalities are most likely included in the code. Afterwards, have them read through the code and check their predictions.

C2.2 Coding

create code by decomposing situations into computational steps in order to represent mathematical concepts and relationships, and to solve problems

Teacher supports

Examples

- **situations that can be decomposed into computational steps:**
    - generating a sequence of numbers to fit specific criteria
    - determining the density of a subset of numbers within a set
    - determining whether a relationship is linear or non-linear

- generating output values for given input values for a linear relation
- determining the proportionality of two volumes
- examining the effects on volume of changing the size of one of the dimensions
- determining the size of payments on a loan given different interest rates

Instructional Tips

Teachers can:

- embed coding tasks across strands to support students in making sense of mathematical concepts;
- make connections to prior learning in coding from the elementary grades, such as using loops, conditional statements, and subprograms;
- create and facilitate opportunities for students to work in teams to decompose situations and support one another in working through challenges;
- provide students with sections of code to incorporate in order to support their understanding of the process of decomposing situations;
- support students in refining their algorithms with more efficient steps when decomposing situations;
- model the process of solving a smaller problem for a specific case, then generalizing it to solve for multiple cases;
- demonstrate ways in which subprograms can be helpful when decomposing situations;
- provide opportunities for students to share their approaches to solving a problem through coding in order to appreciate the various ways in which it can be solved.

Note

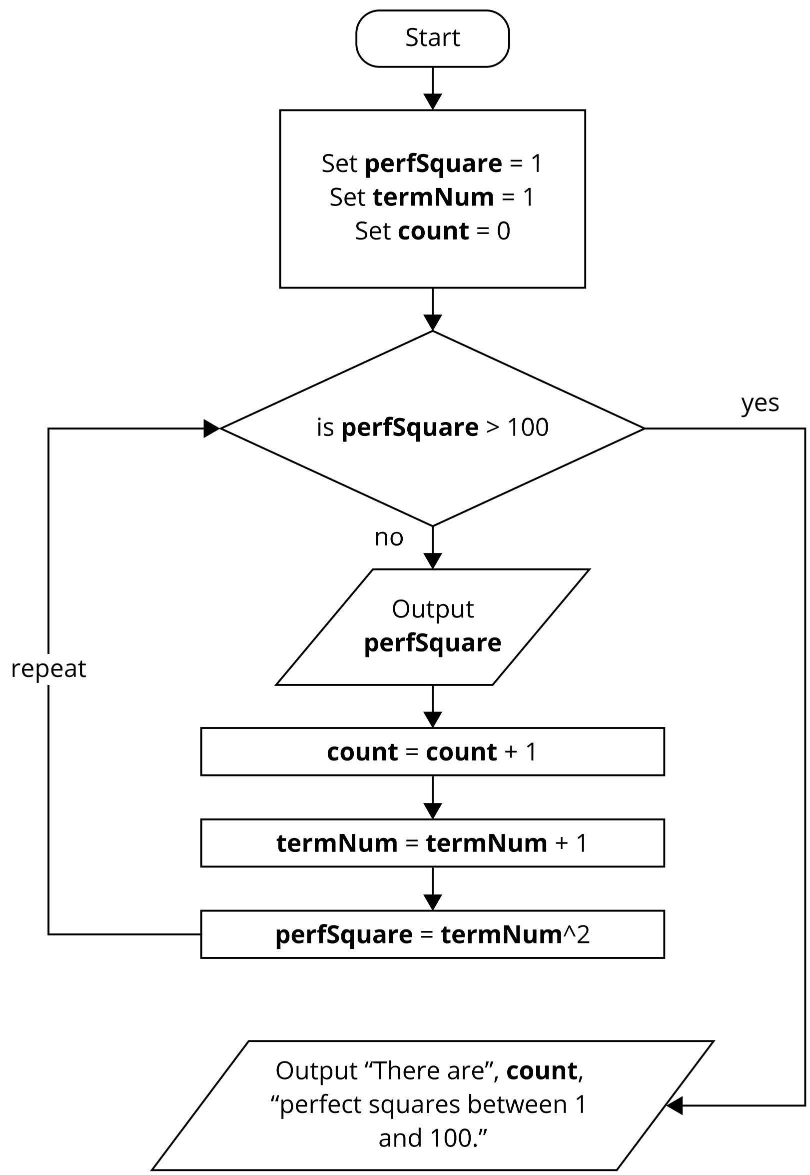
The learning in C2.1, C2.2, and C2.3 is interconnected, and this should be reflected during instruction.

Teacher Prompts

- What different components of this problem do you need to consider in order to solve it?
- What coding structures (e.g., conditional statements, repeating events) will help you solve this problem?
- In this block of code, is the order of the steps important? What would happen if we put the same steps in a different order? Would it change the outcome of the program?

Sample Tasks

1. Have students use the logic in a flow chart to write code to represent a mathematical concept that they are learning. For example, show them the flow chart below, and explain how following the steps would generate the set of perfect squares from 1 to 100. Have them write, execute, and edit their own code until the desired outcome is achieved. Next, have them alter their code so that the program determines the perfect squares from 1 to 500, or from 1 to 1000. As an alternative, have students alter their code so that the program determines the perfect cubes from 1 to 100, or from 1 to 500.



2. Have students work collaboratively to create a flow chart to show the steps needed to determine whether a relation is linear or non-linear. Then have them write the code using a text-based or block-based program. Finally, have them execute the code and identify areas in the flow chart that need to be altered if the executed code did not produce the desired outcome.

3. Provide students with a problem that can be solved using code.

   - Have them work in small groups to identify the steps needed to solve the problem.
   - Ask individual students to create a flow chart outlining their steps.
   - Have them compare their flow charts within their group, identifying the similarities and differences.
   - Then have them create pseudocode from their flow charts. (Note: As students create their pseudocode, they may need to adjust their flow chart.)
   - Have them use their pseudocode to write, execute, and adjust a text-based or block-based program until they get the desired outcome.

C2.3 Coding

read code to predict its outcome, and alter code to adjust constraints, parameters, and outcomes to represent a similar or new mathematical situation

Teacher supports

Examples

- **predicting outcomes in order to:**
  - visualize the mathematics involved
  - ensure that the code will execute properly
  - deconstruct code to understand its purpose and meaning
- **altering code in order to:**
  - simplify the code
  - debug the code to produce desired outcomes
  - solve similar problems
  - produce different outputs
  - apply the code to a new mathematical situation

Instructional Tips

Teachers can:

- facilitate and create opportunities for students to work in teams or pairs to solve problems and support one another in working through challenges;

- use reading and altering code as a starting point for supporting students in coding, perhaps progressing to students writing their own code later on;
- use reading and altering code as tools to enhance students' mathematical learning;
- choose a coding language that students are familiar with from prior experience or connect a new coding language to one they are already familiar with;
- acknowledge, and encourage the sharing of, the different knowledge and experiences that students bring to coding.

Note

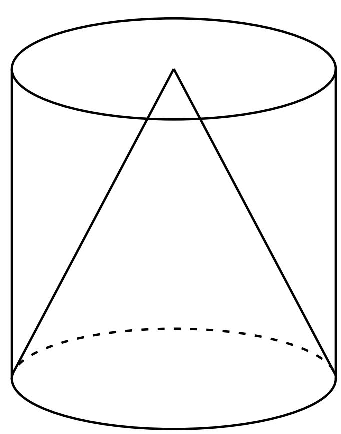
The learning in C2.1, C2.2, and C2.3 is interconnected, and this should be reflected during instruction.

Teacher Prompts

- What strategies do you use when reading code to determine its outcome?
- What happens to each variable as the code is executed? Is this the result you expected? If not, why not?
- When altering this code, what will you keep the same, and what will need to change?
- If we complete some of the steps in a given code in a different order, will we get a different output?

Sample Tasks

1. Provide students with sample code, pseudocode, or a flow chart, and ask them to predict the outcomes. For example, provide them with the flow chart and pseudocode below to answer the following problem: What is the amount of empty space in the cylinder that is not occupied by the cone?

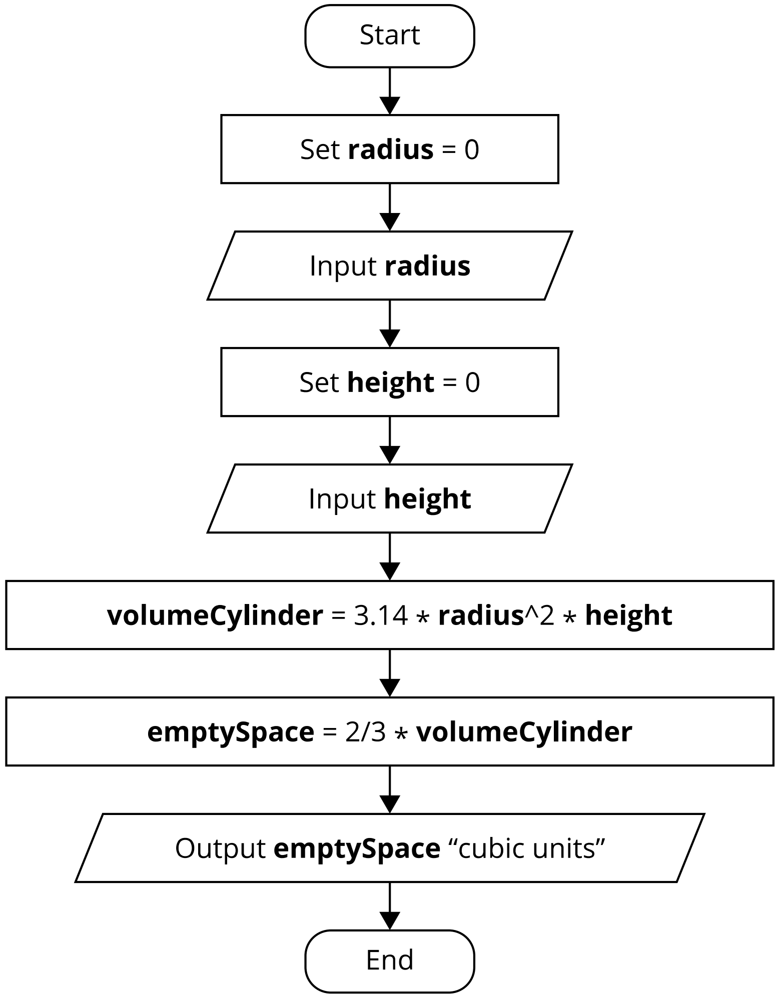


Then ask:

- What is the same and what is different between the pseudocode and the flow chart?
- Do the pseudocode and the flow chart have the same outcome? Explain why or why not.

emptySpace = 0
volumeCylinder = 0
volumeCone = 0
height = 0
radius = 0
output “Enter the height of the cylinder and the cone.”
store user input as **height**
output “Enter the radius of the circular base of the cylinder and the cone.”
store user input as **radius**
volumeCylinder = π * **radius**^2 * **height**
volumeCone = **volumeCylinder**/3
emptySpace = **volumeCylinder** − **volumeCone**
output “The empty space in the cylinder that is not occupied by the cone is”, **emptySpace**, “cubic units.”

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.



2. Provide students with pseudocode, and ask them to alter the code for a new situation. For example, have them alter the code below to determine the volume of a cone, using their understanding of the relationship between the volume of a cone and the volume of a cylinder.

radius = 0
height = 0
volumeCylinder = 0
output "What is the radius of the cylinder?"
store user input as **radius**
output "What is the height of the cylinder?"
store user input as **height**
volumeCylinder = π * **radius**^2 * **height**
output "The volume of the cylinder is", **volumeCylinder**, "cubic units."

3. Provide students with sample code that calculates the sum of the first five terms in the sequence $\frac{1}{2}$, $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$. Then have them alter the code to add more terms and explore the limit of this sum.

4. Provide students with sample code that finds the least value in a set of data, and have them alter the code to find the greatest value.

5. Provide students with sample code that determines the amount of interest earned each month for an investment earning simple interest. Have them alter the code for an investment earning interest that is compounded monthly.

Overall expectation

C3. Application of Relations

represent and compare linear and non-linear relations that model real-life situations, and use these representations to make predictions

Specific expectations

By the end of this course, students will:

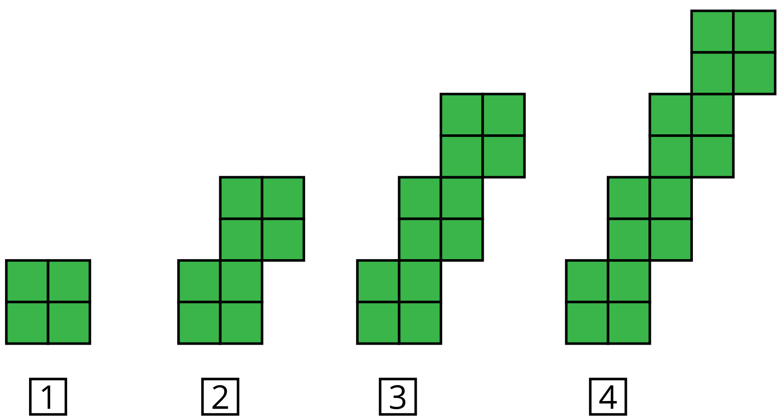
C3.1 Application of Linear and Non-Linear Relations

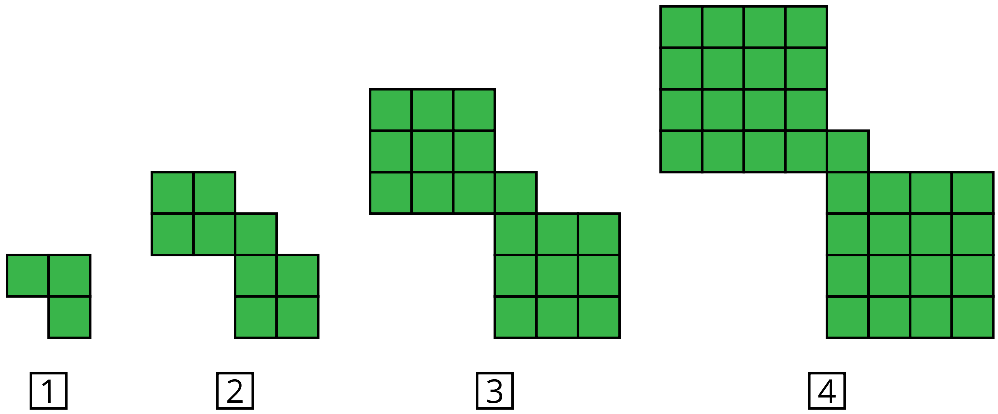
compare the shapes of graphs of linear and non-linear relations to describe their rates of change, to make connections to growing and shrinking patterns, and to make predictions

Teacher supports

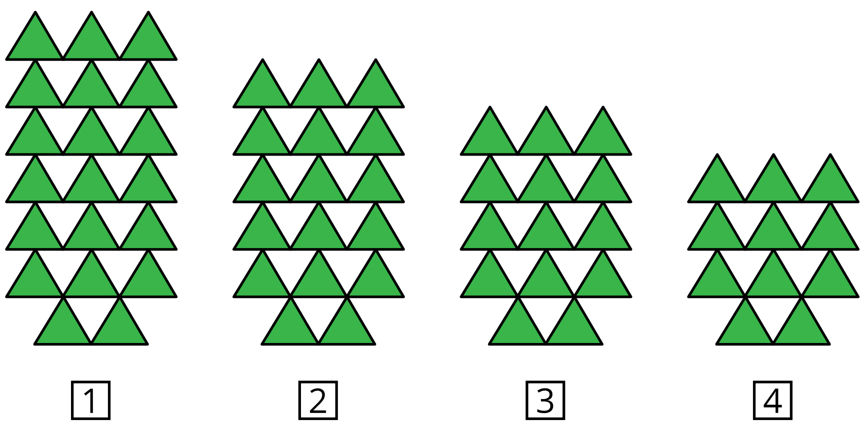
Examples

- **real-life situations involving linear and non-linear relations:**
  - the volume of fruit juice that can be made in comparison to the mass of fruit
  - the total amount earned in comparison to the number of hours worked
  - the volume of water in a pool over time as it is being emptied or filled
  - the number of times a piece of paper is folded in half compared to the number of layers of paper after each new fold
  - the monetary value of a vehicle over time
  - the height that a basketball rebounds after each bounce
- **describing rates of change:**
  - constant rate of change
  - zero rate of change
  - positive or negative rate of change
  - increasing rate of change
  - decreasing rate of change
- **growing patterns:**





- **shrinking patterns:**



Instructional Tips

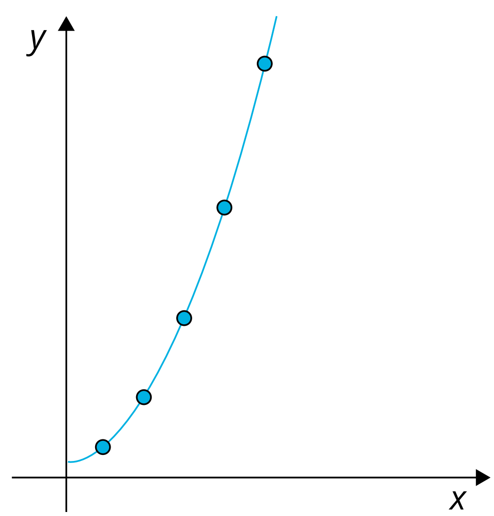
Teachers can:

- pose situations to students and facilitate a conversation about what the shape of the graphical representation of each situation tells us about the situation;
- provide students with concrete materials and digital tools to build linear and non-linear growing and shrinking patterns, and support them in creating graphs to represent these patterns;
- support students in making connections between the shapes of the graphical representations and their rates of change (e.g., if the graph becomes steeper, what does that tell us about the rate at which the variables are changing?);
- encourage students to describe graphs and their rates of change using gestures, vocabulary, and other means that are accessible to them, along with mathematical terminology;

- share graphs that have connections to other strands in the course (e.g., appreciation, changing volume);
- facilitate a discussion about what strategies students might use to make predictions (e.g., interpolating and extrapolating) and when it is appropriate to use their graphs to make predictions.

Teacher Prompts

- How can you tell from the shape of a graph when it:
  - is growing at a constant rate?
  - is growing at an increasing rate?
  - is growing at a decreasing rate?
  - has a rate of change of zero?
  - is shrinking at a constant rate?
- Explain how the shape of a graph can provide information about the situation it represents. How can the graph help you make predictions about the relationship?
- What are the differences between linear and non-linear relations? What are the differences in their graphs? In their rates of change?
- If you built a pattern to represent the graph below, what would it look like? Would the pattern be growing or shrinking as the independent variable increases? Would it go up by a constant amount or by a changing amount? How could you use the graph to make a prediction about the pattern?



Sample Tasks

1. Have students create a growing or shrinking pattern with objects of their choice and then make a graph to represent their pattern. Post images of the patterns and the graphs, and ask students to

match them by comparing how the patterns are changing with the direction and shape of the graphs. Ask them to justify their choices. Pose questions that require students to make predictions about the future behaviour of the patterns, such as “Which pattern will have a higher value for term 10?”

2. Show students various graphs, and ask them to predict how they would need to walk in front of a motion detector to create the graph. For example, students might say “I would walk away from the motion detector very slowly at first and then speed up.” If possible, have them test their predictions with motion detectors.

3. Ask students to sketch a graph for situations either described in words or shown on video. Then ask them to describe the rates of change in the graphs and how they connect to the situations represented. Some examples of situations are:

    - the position of a person over time when running at various speeds
    - the height of a person over time on various pieces of playground equipment or amusement park rides
    - the population of bacteria over time when the population keeps doubling
    - the height of a person over their lifetime
    - the value of a car over time after it is purchased
    - the height of a basketball after each bounce
    - the average daily temperature over a year

4. Have students compare the graph of an investment growing with simple interest to the graph of an investment growing with compound interest. Have them use the graphs to predict the value of the investment at various points on the graph (interpolation) and at points beyond the given graph (extrapolation).

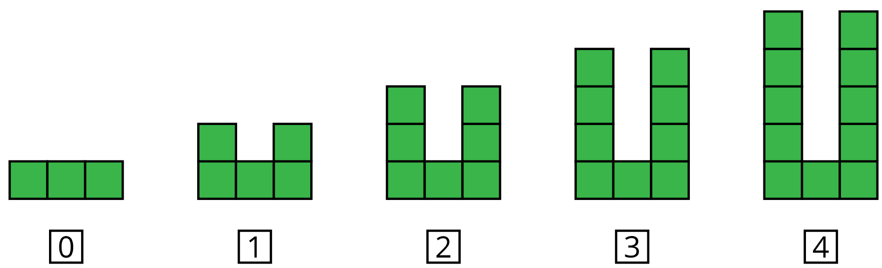
C3.2 Application of Linear and Non-Linear Relations

represent linear relations using concrete materials, tables of values, graphs, and equations, and make connections between the various representations to demonstrate an understanding of rates of change and initial values

Teacher supports

Examples

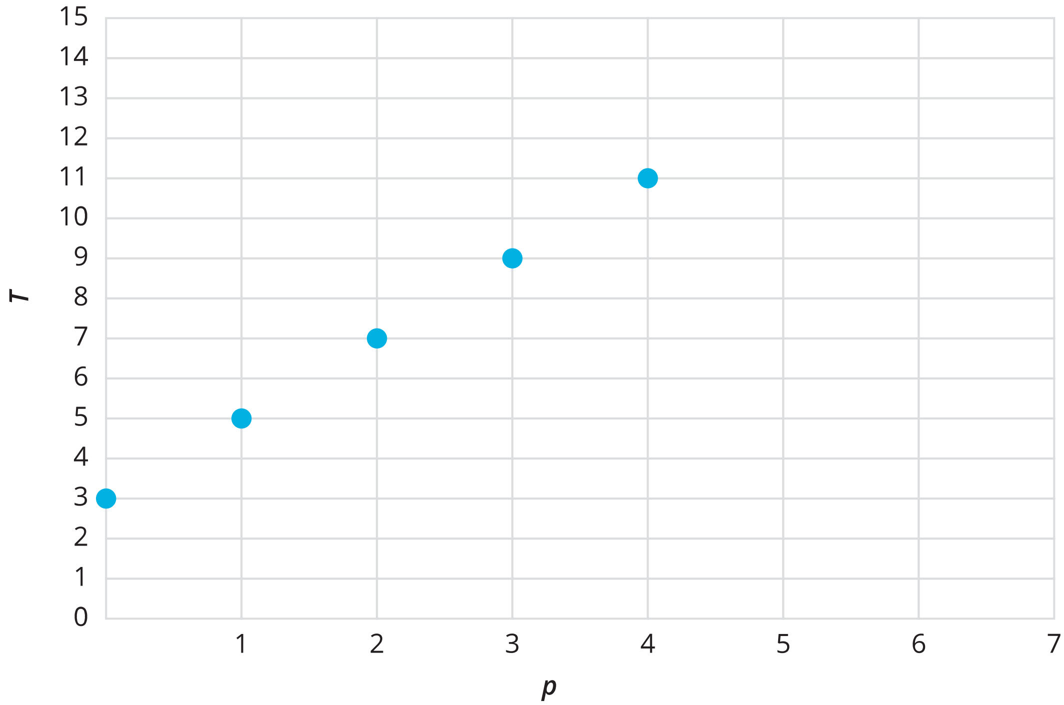
- **linear relations represented using:**
  - concrete materials:



  - tables of values:

position number (term number)	number of tiles (term value)
0	3
1	5
2	7
3	9
4	11

  - graphs:



- equations:
  number of tiles = 2 × (position number) + 3
  or
  $T = 2p + 3$
  or
  NumTiles = 2*PosNum + 3

Instructional Tips

Teachers can:

- ground the learning for this expectation in real-life examples and contexts that are relevant to students;
- provide students with opportunities to use concrete materials (e.g., colour tiles, interlocking cubes, cups, pine cones, beads), digital tools, and coding to represent linear relations;
- support students in developing ways to determine the rate of change and the initial value in each of the various representations;
- provide students with opportunities to make connections between representations by providing them with one or two representations and supporting them in generating the others;
- facilitate a discussion about which representations are most helpful for various purposes, including making and reflecting on both near and far predictions;

- continue to make connections between the rate of change and the initial value and what they represent in a given context;
- introduce the terms *partial variation* and *direct variation* as connected to initial values and proportionality;
- encourage students to identify relationships using functional thinking by making connections between the term value (dependent) and corresponding term number (independent) as well using recursive thinking by making connections from term value to term value.

Teacher Prompts

- How can you determine the rate of change from each representation? Do you find it easier to determine the rate of change with some types of representations than with others?
- How can you determine the initial value from each representation? Do you find it easier to determine the initial value with some types of representations than with others?
- Given one of the four types of representations (concrete materials, table of values, graph, and equation), how do you create the others for the same relation?
- Given four representations of different types, how do you know they all represent the same relation?
- What does the rate of change mean in this context? What does the initial value mean in this context?

Sample Tasks

1. Provide students with a set of 20 cards (concrete or digital). Each card should have one of five different linear relations, each represented in one of four ways: visual, table of values, graph, and equation. Have them match sets of cards that show different representations of the same linear relation. For an added challenge, replace some of the representations with blank cards for students to complete.

2. Provide students with one representation of a linear relation in context, and ask them to create a different representation of the relation. Some examples of contexts that might be relevant to students' lives are:

   - cost of participating in various classes (e.g., dance, yoga, martial arts, fitness, music)
   - distance travelled over time
   - number of hours worked and total pay
   - mass of bulk goods purchased and cost
   - area of land and crop yield

3. Have students create one representation of a linear relation that illustrates a scenario they have created. Then have them trade representations, describe the scenario, and create a different representation of it.

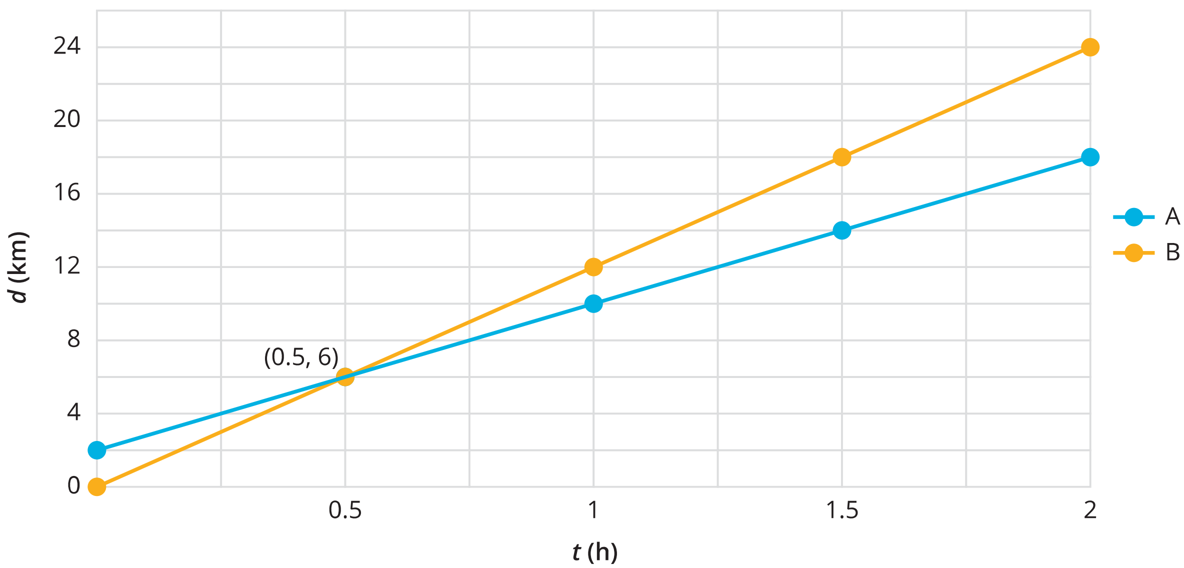
C3.3 Application of Linear and Non-Linear Relations

compare two linear relations of the form $y = ax + b$ graphically and algebraically, and interpret the meaning of their point of intersection in terms of a given context

Teacher supports

Examples

- **compare graphically:**
  - create a graph comparing distance in kilometres and time in hours:



  - compare rates of change, initial values, and point of intersection
- **compare algebraically:**
  - compare rates of change and initial values by examining the equations
  - use the method of comparison to compare relation A: $d = 8t + 2$ to relation B: $d = 12t$ by setting the equations equal to each other and solving for the variables to determine the point of intersection

Instructional Tips

Teachers can:

- sequence the learning to begin with two relations that allow students to accurately locate the point of intersection on a graph and move to relations that require an algebraic method of comparison to accurately determine the point of intersection;

- facilitate a discussion about the strengths and limitations of comparing relations graphically versus comparing them algebraically;
- support students in interpreting the point of intersection in the context of the problem by highlighting that this is the point that satisfies both equations and the point where, for a specific independent value, both relations have the same dependent value;
- provide students with opportunities to use digital tools or coding to compare linear relations;
- support students in making connections between the algebraic method of comparison that they are learning now and the strategies they learned for solving equations with variables on both sides of the equal sign in Grade 8.

Note

The use of the algebraic method is intended to involve only the method of comparison, not to be extended to elimination, substitution, or other approaches to solving linear systems.

Teacher Prompts

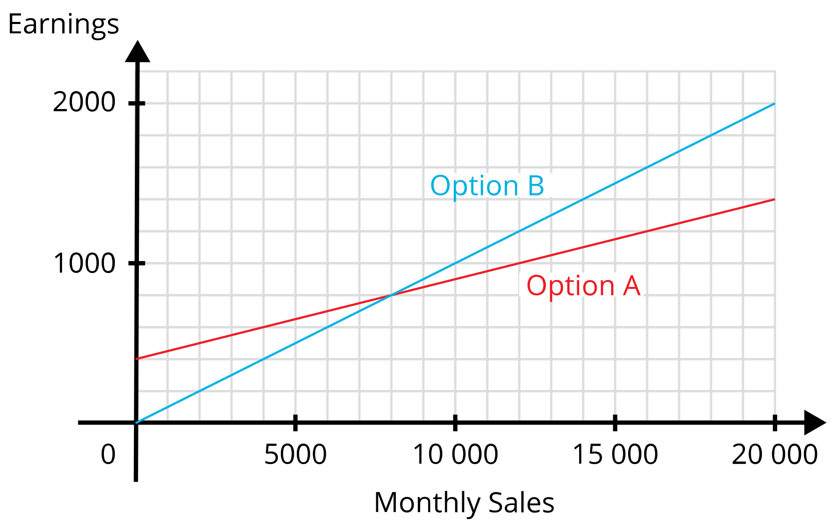
- What is similar and what is different about these two linear relations?
- What does the point of intersection (if any) mean in this situation?
- Compare the rates at which these relations are growing.
- Thinking about the context of a situation shown graphically, describe what is happening on the left and right sides of the point of intersection (if any) on the graph.
- List the strengths and weaknesses of both the graphical method and the algebraic method of comparing relations.
- Will two linear relations always have a point of intersection? In what cases will they not intersect?

Sample Tasks

1. Have students compare two different linear relations and identify when they are equal (the point of intersection, if they have one) and when one is greater than the other. Some possible contexts are: pages read in a book, earnings from a job, cell phone plans, distance travelled, gym memberships.

   For example: A salesperson has two different options for getting paid. The options are shown on the graph below. What should they consider when choosing the best option for them? What are the differences between the two options? Under what conditions would each option be better?

   - Option A: A monthly salary of $400 plus a 5% commission on total monthly sales
   - Option B: No set monthly salary, but with 10% commission on total monthly sales



2. Have students compare relations where an algebraic solution might allow for more accuracy. For example: During a professional car-racing event, two drivers are jockeying for first place. Driver A, currently in first place, has a 15 second lead over Driver B, in second place, but Driver B has picked up speed. The following equations can be used to represent the amount of time, t, in minutes, that they have been driving since Driver B picked up speed, in terms of n, the number of laps.

   - Driver A: $t = 1.4n$
   - Driver B: $t = 1.3n + 0.25$

   At what point will Driver B catch up to Driver A? If there are five laps left in the race, is it possible for Driver B to win?

Overall expectation

C4. Characteristics of Relations

demonstrate an understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate

Specific expectations

By the end of this course, students will:

C4.1 Characteristics of Linear and Non-Linear Relations

compare characteristics of graphs, tables of values, and equations of linear and non-linear relations

Teacher supports

Examples

- **characteristics:**
  - **of graphs:**
    - shape
    - direction or orientation
    - intercepts
    - rate of change (constant or not constant)
    - symmetry
  - **of tables of values:**
    - initial value
    - rate of change (constant or not constant)
    - symmetry
    - patterns or repetition
  - **of equations:**
    - degree
    - leading coefficient when terms are in descending order of powers
    - type of equation (e.g., $y = x^2$ (quadratic), $y = 2^x$ (exponential), $y = 2x$ (linear), $y = \frac{1}{2x}$(reciprocal))
    - rate of change (constant or not constant)
    - initial value

Instructional Tips

Teachers can:

- encourage students to develop and expand their vocabulary for describing, explaining, and making connections among the characteristics of graphs, tables of values, and equations;
- facilitate discussions to highlight comparisons and connections between different representations of the same relation as well as between the same representation of different relations;
- support students in recognizing how and why calculating first differences in tables of values can be helpful for determining rates of change and identifying linear and non-linear relations;
- provide opportunities for students to use coding or digital tools to create graphs.

Teacher Prompts

- How can you determine whether a relation is linear or non-linear by examining the graph? By examining the table of values? By examining the equation?
- Which of these graphs have a line of symmetry?

- What are the x- and y-intercepts of this graph, if any? Why does it make sense that the graph intersects the axes in these places?
- How does the graph of $y = 2x$ compare to the graph of $y = -2x$?
- How are the graphs of $y = 2x$, $y = x^2$, and $y = 2^x$ similar, and how are they different? How do these similarities and differences appear in their tables of values and in their equations?

Sample Tasks

1. Have students use graphing technology to make graphs of various relations and to make note of characteristics that they notice in each graph.

2. Provide students with a set of cards, each showing the graph, the table of values, or the equation of a linear or non-linear relation. Have them sort the graphs according to different characteristics. For example:

   - those with a constant rate of change and those with a changing rate of change
   - those that are always increasing, those that are always decreasing, those that both increase and decrease
   - those that have the same y-intercept

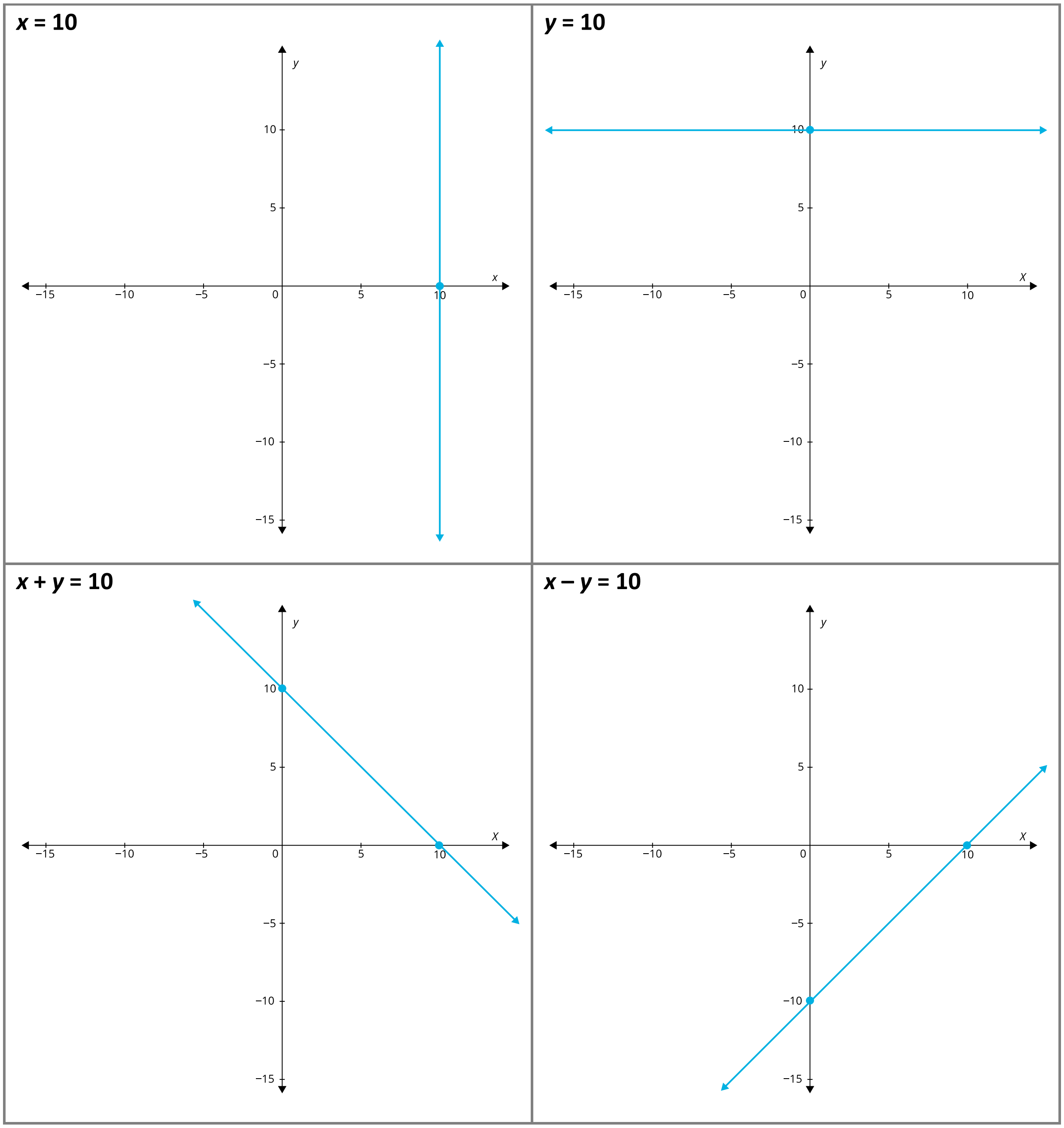
C4.2 Characteristics of Linear and Non-Linear Relations

graph relations represented as algebraic equations of the forms $x = \text{k}$, $y = \text{k}$, $x + y = \text{k}$, $x - y = \text{k}$, $\text{a}x + \text{b}y = \text{k}$, and $xy = \text{k}$, and their associated inequalities, where a, b, and k are constants, to identify various characteristics and the points and/or regions defined by these equations and inequalities

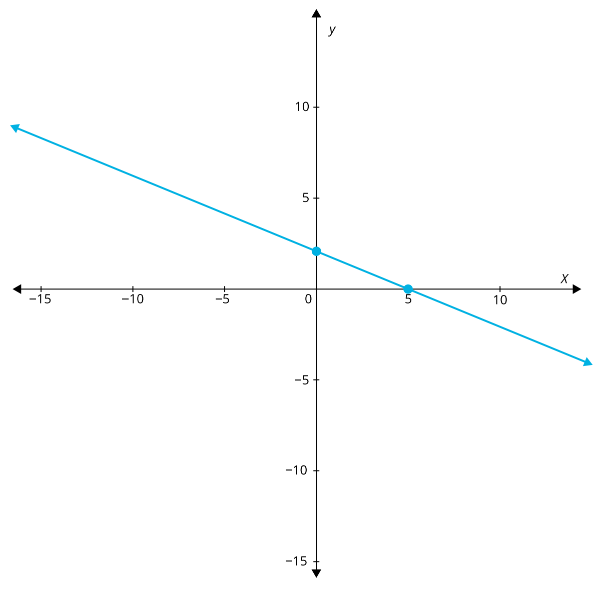
Teacher supports

Examples

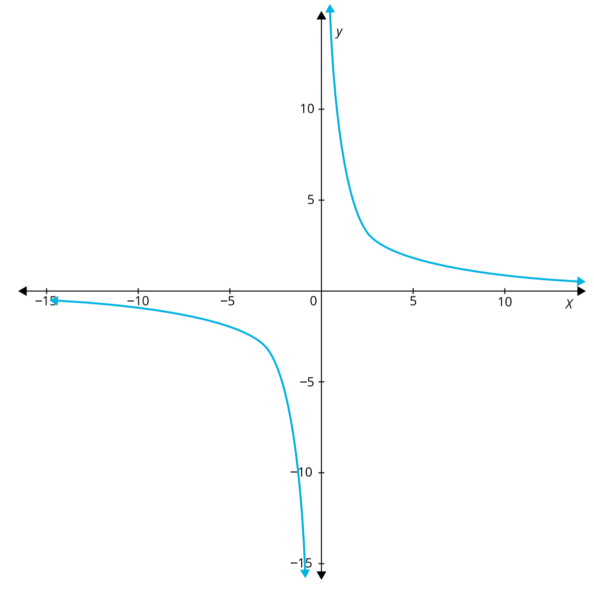
- **graphs of relations:**


x = 10
y = 10
x + y = 10
x − y = 10
x
y
−15
−10
−5
0
5
10

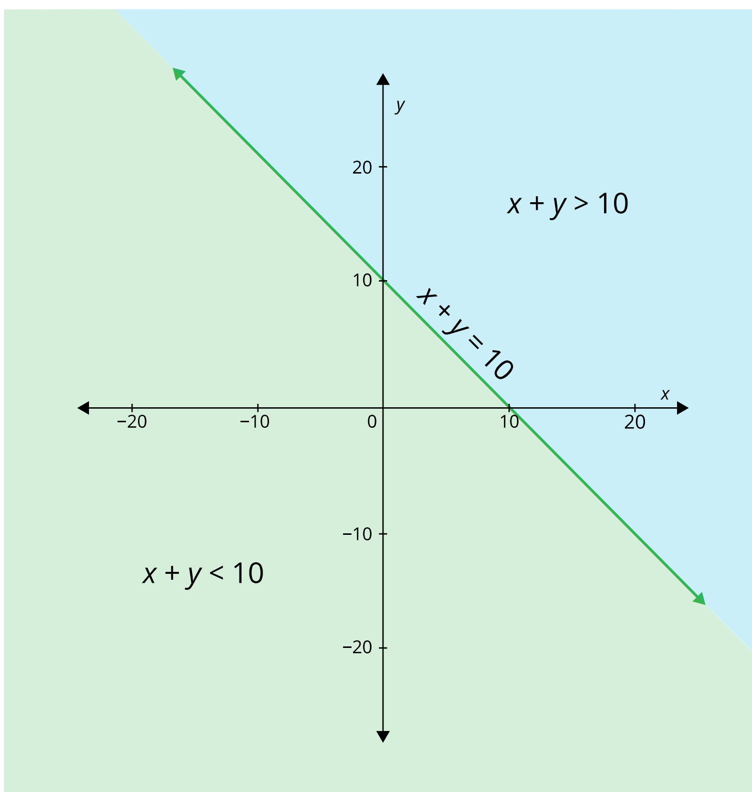
$2x + 5y = 10$



$xy = 10$



- **associated inequalities:**



Instructional Tips

Teachers can:

- create opportunities to reinforce math facts by asking students to list possible ordered pairs that make an equality true (e.g., for the relation $x + y = 10$, ask "What are some pairs of numbers that have a sum of 10?");
- support students in moving from thinking about discrete points that satisfy equations and inequalities to thinking about continuous lines or regions and making connections to the concept of density of numbers;
- depending on student readiness, lead conversations that require students to reflect on the characteristics of different graphs, depending on the form of the algebraic equation. This will support them in building questions they can ask themselves to make conjectures about what a graph might look like; for example:
    - Where would the graph of $x + y = 10$ intersect the x-axis? the y-axis? How do you know?
    - If two numbers, x and y, multiply to a positive number (i.e., $xy = k$, where $k > 0$), what has to be true about x and y? What would this look like in a graph? In which quadrants would you find points on the graph? Explain your thinking.
- encourage students to make conjectures about what a graph will look like by first sketching the graph and then using coding or digital tools to test their conjectures;
- highlight $x = k$ and $y = k$ as special cases of linear relations, and have students explore why they are special cases;
- support students in noticing how y-values change, depending on linear and non-linear relationships between x and y (e.g., ask: What do we know about y if $xy = 10$? if $x + y = 10$?);
- introduce students to the different ways of representing inequalities on a graph (e.g., if showing the region $x + y < 10$, the line for $x + y = 10$ should be dotted as opposed to solid);
- support students in developing strategies for determining whether a given point satisfies an equation or inequality represented on a graph, and how this point connects to various regions on the graph;
- facilitate opportunities for students to explore, using coding and digital tools, multiple cases of each of the equations and associated inequalities listed above in order to highlight their characteristics.

Teacher Prompts

- Think of two numbers that have a sum of 10. Do only whole numbers work? What integer values work? What about fractions or decimals?
- How does $x = 10$ compare to $x > 10$? What numbers satisfy $x = 10$? What numbers satisfy $x > 10$?
- What numbers satisfy $x + y > 10$? Where do these points lie on the grid in comparison to the line $x + y = 10$?
- Think of two numbers that can be multiplied to get 180. Do only whole numbers work? What integer values work? What about fractions or decimals?

- What do you notice about where each of these graphs crosses the *x*-axis? The *y*-axis? Why does it make sense that they cross at these points?

Sample Tasks

1. Ask students to generate a set of coordinates that satisfy the equation $x + y = 10$. Have them plot these coordinates on a grid, and then discuss whether they have found all the possible values that satisfy the equation or if there are others between these points. This discussion should lead to the idea of connecting the points with a line to represent all possible values. Then have them choose points above and below the line they have drawn, and ask them how these points are connected to the inequalities $x + y > 10$ and $x + y < 10$.

2. Have students choose values for *k*, *a,* and *b* to explore, and then graph, using technology, the relations $x = k$, $y = k$, $x + y = k$, $x - y = k$, $ax + by = k$, and $xy = k$ and compare them using the characteristics discussed in class. Then have them use technology again to explore the inequalities for these relations.

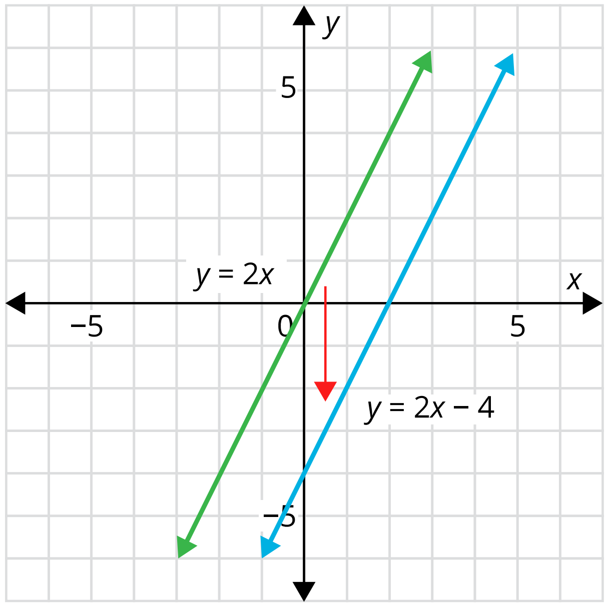
C4.3 Characteristics of Linear and Non-Linear Relations

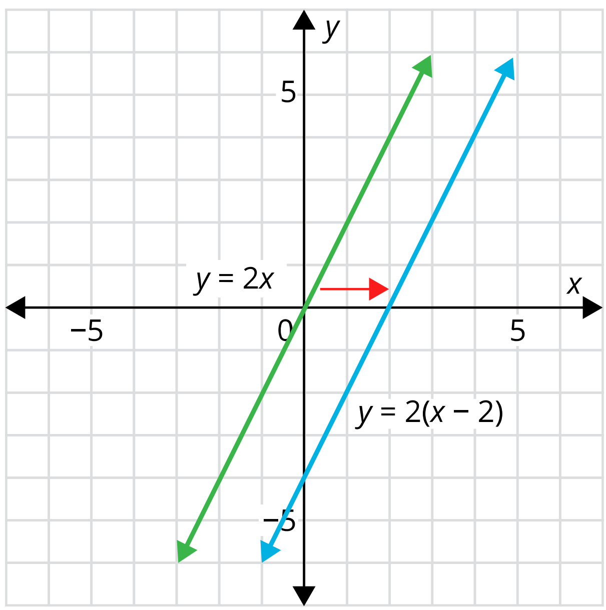
translate, reflect, and rotate lines defined by $y = ax$, where a is a constant, and describe how each transformation affects the graphs and equations of the defined lines

Teacher supports

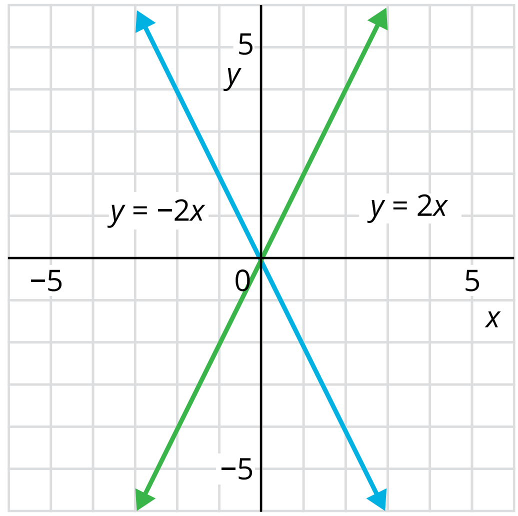
Examples

- **transformations:**
  - **translations:**
    shift up, down, left, or right:

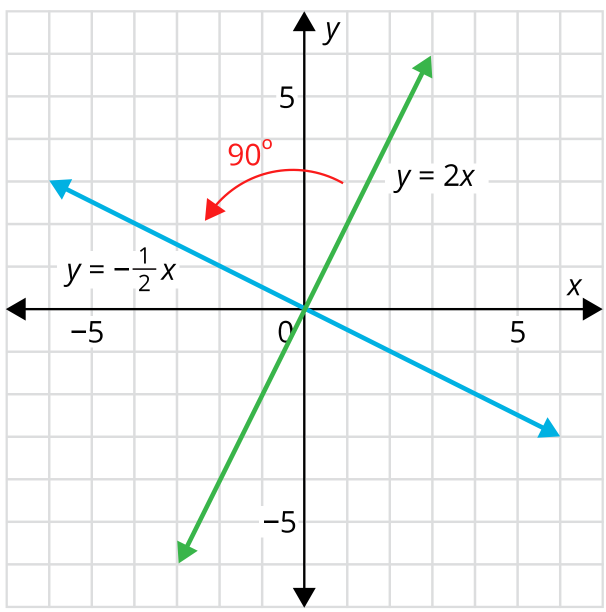

y
5
y = 2x
x
−5
0
5
y = 2x − 4
−5


y
5
y = 2x
x
−5
0
5
y = 2(x − 2)
−5

- **reflections:**
  over the *x*-axis or over the *y*-axis:



- **rotations:**
  clockwise or counterclockwise in increments of 90°:



Instructional Tips

Teachers can:

- make connections between learning from the elementary grades about transformations of points and shapes and the transformation of lines by providing students with graphs of lines (physical or digital) and asking them to perform the various transformations on them;
- support students in making connections between the changes in the equations and the changes in the graphs;
- provide opportunities for students to use coding or digital tools to test conjectures about how different transformations will affect the graph of a line and how changing the equation of the line will affect the graph;
- make connections to C1.3 when examining the effects of a translation up/down and a translation left/right; for example, point out that a shift down 4 ($y = 2x - 4$) has the same effect as a shift right 2 ($y = 2(x - 2)$), and ask "How are the equations similar and how are they different?";
- support students in making connections between the y-intercepts and slopes of the lines and the equations of the lines;
- facilitate a discussion about the relationship between the equations of parallel lines when looking at translations and rotations of 180° or 360° and the relationship between the equations of perpendicular lines when looking at rotations of 90° or 270°;
- introduce the concept of combining transformations, depending on student readiness.

Note

The learning in this expectation connects learning from the elementary grades about transformations of points and shapes to the transformation of lines. This connection will then support learning about the transformation of more complex functions in future grades.

Teacher Prompts

- How does translating, reflecting, and rotating a line connect to transforming shapes?
- What changes do you notice in this transformed graph? How is it the same? How is it different?
- If you start with the line $y = 2x$, what type of transformation would result in:
    - a parallel line with a y-intercept of 4?
    - a parallel line with an x-intercept of −3?
    - a line that is perpendicular to $y = 2x$?
    - a line that has a negative slope and the same y-intercept as $y = 2x$?
- Notice that a translation of 4 units down of $y = 2x$ has the same effect as translating it 2 units right. How are the equations that represent the line after these transformations similar, and how are they different?
- What do you notice about the slope of a graph that is rotated 90° about the origin compared to the slope of the original graph?

- What do you notice about the slope of a graph that is rotated 180° about the origin compared to the slope of the original graph?

Sample Tasks

1. Provide students with the graph of a line $y = ax$ for a given value of a on a grid. Provide tools such as tracing paper, rulers, and Miras, and have students perform translations, rotations, and reflections on the graph and describe how each transformation affects the graph.

2. Have students use graphing technology to explore the effects of various transformations on the line $y = ax$ for a chosen value of a. Have them explore which transformations are connected to each of the following changes in the equation:

   - $y = ax + b$
   - $y = a(x - c)$
   - $y = -ax$
   - $y = -\frac{1}{a}x$

3. Have students rotate the line $y = ax$, for a chosen value of a, clockwise or counterclockwise by 90°. Have them compare the slopes of these lines and discuss how the slopes of perpendicular lines are related. Their rotation might look something like the animation below:

4. Have students translate the line $y = ax$ up or down for a chosen value of a. Have them compare the slopes of these lines and discuss how the slopes of parallel lines are related.

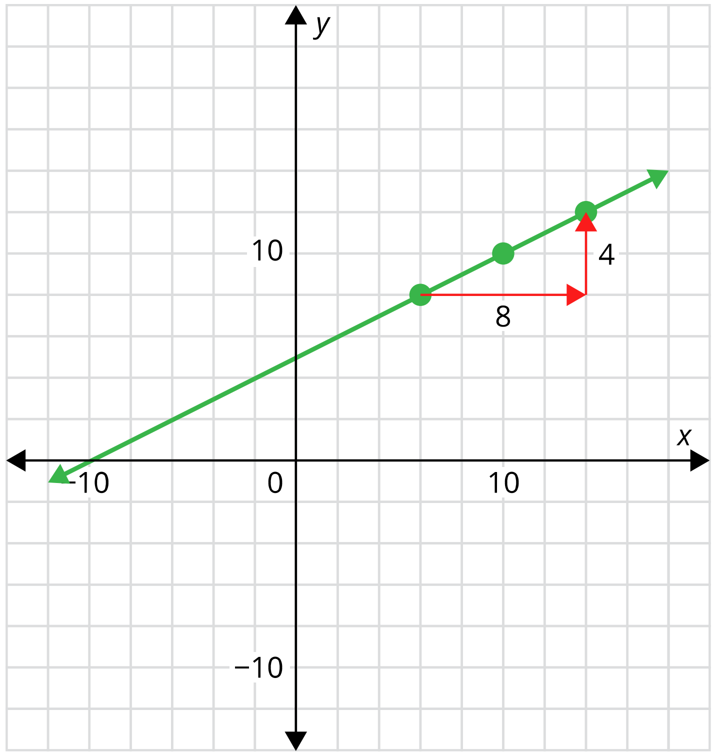
C4.4 Characteristics of Linear and Non-Linear Relations

determine the equations of lines from graphs, tables of values, and concrete representations of linear relations by making connections between rates of change and slopes, and between initial values and y-intercepts, and use these equations to solve problems

Teacher supports

Examples

- **determining the equation of lines from representations:**
  - **from a graph:**



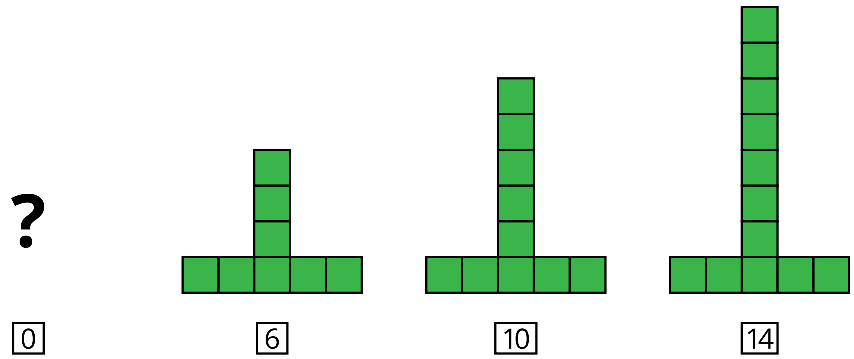
- the slope (rate of change) is $\frac{4}{8} = \frac{1}{2}$
- the line intersects the *y*-axis at 5, so the *y*-intercept (or initial value) is 5
- an equation for this relationship is $y = \frac{1}{2}x + 5$

- **from a table of values:**

	x	*y*	
	6	8	
+ 4			+ 2
	10	10	
+ 4			+ 2
	14	12	

- the rate of change (slope) is $\frac{2}{4} = \frac{1}{2}$ since the dependent values increase by 2 as the independent values increase by 4
- by using the relationship and working backwards through the table, students can determine the initial value (or *y*-intercept) of 5
- an equation for this relationship is $y = \frac{1}{2}x + 5$

- **from a concrete representation:**



- there are 2 more tiles every time the position number increases by 4, so the rate of change (slope) is $\frac{2}{4} = \frac{1}{2}$
- working backwards and using the relationship between the position number and the tiles, it can be determined that position 0 has 5 tiles
- an equation for this relationship is $y = \frac{1}{2}x + 5$

Instructional Tips

Teachers can:

- pose problems that support students in moving from relations in the first quadrant to those that involve other quadrants;
- support students in making connections between the rate of change and the slope of the graph and between the initial value and the *y*-intercept by using concrete, numerical, and graphical representations;
- pose problems, including those involving relevant real-life contexts, that require students to predict values beyond the given information, and within the given information, by using graphs, tables, concrete representations, and equations;
- facilitate a discussion about the strengths and limitations of each representation (e.g., if the rate of change is fractional, it may not be possible to build every position of the concrete representation; if the values are very large, it may be hard to recognize them on a graph);
- encourage students to represent a relationship in a different way in order to work with a representation that they may be more comfortable with;
- support students in developing strategies for determining the rate of change from various representations and for various situations (e.g., fractional rates of change, negative rates of change, different scales on the *x*- and *y*-axes).

Teacher Prompts

- How does the number of tiles added each time in a concrete pattern relate to the slope of the graph representing this relation?
- How can you determine the slope of a line from a graph? From a table of values?
- What information do you need to determine the equation of a line?
- Why can you use any two points on a line to determine the slope?
- What happens to this linear pattern if we move backwards from term 2 to term 1 to term 0? What might term −1 look like? How would you represent it on a graph?

Sample Tasks

1. Have students use concrete materials to build several terms of a linear pattern and write the general rule for the pattern. Then have them plot the given terms of the pattern on a graph and use the graph to predict what happens before term 0. Have them make connections between the general rule for the pattern and the rule written in the form $y = ax + b$.

2. Have students determine an equation to represent the relationship between the number of sides of a polygon and the sum of the interior angles of the polygon, using the information in the table below.

Number of Sides	Sum of the Interior Angles
3	180°
4	360°
5	540°

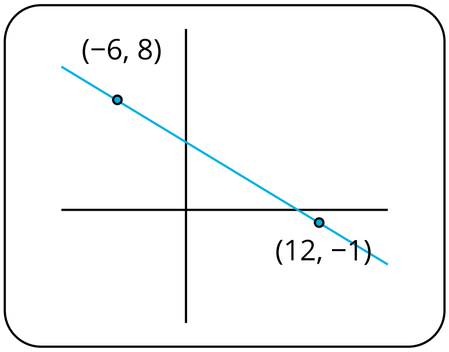
3. Have students plot this relation on a graph, determine the slope of the line, and discuss how the slope is related to the context.

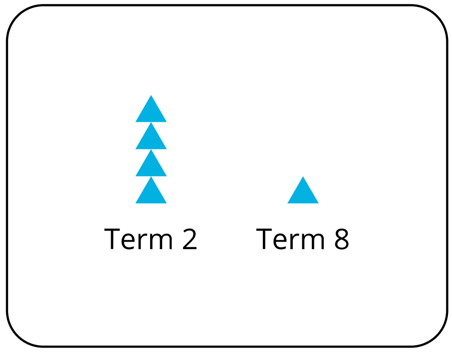
4. Provide students with sets of cards, each showing an equation, a graph, a table of values, or a visual model. Have them match cards that represent the same relation and discuss how the equation connects to the other representations. Include representations where non-consecutive values are given, and some cards that are missing information so that students can complete them.

   A completed set might look like the following:

$$y = -\frac{1}{2}x + 5$$

x	y
−10	10
4	3
8	?





5. Have students use and compare various strategies to determine the equation of a line when given two points on the line. For example, have them determine the equation of the line connecting the points (−6, −10) and (4, 8) by:

   - plotting the points and determining the slope and *y*-intercept from the graph, or
   - calculating the slope and substituting it and a point into $y = ax + b$ to determine the value of *b.*

   Ask students how these methods compare in terms of difficulty and accuracy.

D. Data

Overall Expectations

By the end of this course, students will:

D1. Collection, Representation, and Analysis of Data: describe the collection and use of data, and represent and analyse data involving one and two variables

D2. Mathematical Modelling: apply the process of mathematical modelling, using data and mathematical concepts from other strands, to represent, analyse, make predictions, and provide insight into real-life situations

Overall expectation

D1. Collection, Representation, and Analysis of Data

describe the collection and use of data, and represent and analyse data involving one and two variables

Specific expectations

By the end of this course, students will:

D1.1 Application of Data

identify a current context involving a large amount of data, and describe potential implications and consequences of its collection, storage, representation, and use

Teacher supports

Examples

- **contexts involving large amounts of data:**
  - census data collected by the government
  - personal health data and biometric data
  - personal user data collected when purchasing goods and services at stores or online
  - personal user data collected through social media websites, apps, and the search history of users of online search engines
  - big data used for machine learning
  - data that is centrally collected through fitness websites and apps
  - long-term or large-scale scientific research in climate science or epidemiology and population biology
- **potential implications and consequences:**

- the ability to better assess the need for community programs and services and to enact policy and funding changes
- the ability to develop and evaluate targeted advertising campaigns and evaluate their success and impact
- the ability to refine business intelligence to determine what individuals might be interested in and, accordingly, the content that they are exposed to online
- the ability to make advancements in scientific and technological research and development
- the need for privacy protection and other security aspects of data storage

Instructional Tips

Teachers can:

- facilitate student-led discussions of current contexts that are of interest to them;
- invite students to share their own experiences of potential implications and consequences of data collection, storage, representation, and use, creating an inclusive learning environment where students feel safe sharing;
- highlight the implications and consequences of data collection, storage, representation, and use from the perspectives of individuals, various levels of government, corporations, and community organizations;
- provide opportunities for students to reflect on the ways in which data can be misrepresented and used to mislead audiences.

Teacher Prompts

- What are some everyday situations in which personal data is collected? Who is collecting this data? Who might be using it? How might they be using it?
- How is the importance of individual privacy weighed against the societal value of collecting certain data?
- Social media platforms are constantly collecting data on their users: what you post and posts you look at and like, whom you follow and what topics you search, and even whether you linger on a post while scrolling through your feed. They can often use this data to personalize your experience on their platform and also share this data with third parties that use this data to target ads to you personally. What implications does this have on how you use social media?
- Mapping apps that display traffic information get that information by tracking the phones of the people in traffic. What are some advantages and disadvantages of this method of collecting traffic data?
- What are some issues related to the collection and use of Indigenous data? What could be some possible solutions to those issues?

- How can large amounts of data that have been collected over a long period of time (e.g., data related to climate change) be used to make predictions about the future?

Sample Tasks

1. Have students examine how cryptocurrencies use large server farms to collect, store, and mine information. Have students describe implications and consequences of these large server farms, including the amount of energy they consume.

2. Have students discuss how biometric data is captured and used to identify people using facial recognition software. Have students describe implications and consequences of the collection, storage, and use of biometric data by various companies and organizations.

3. Have students investigate the sources of data used to monitor the effects of climate change. Have students discuss how this data is being used to make predictions about future scenarios, and how these scenarios could inform the actions that people take now.

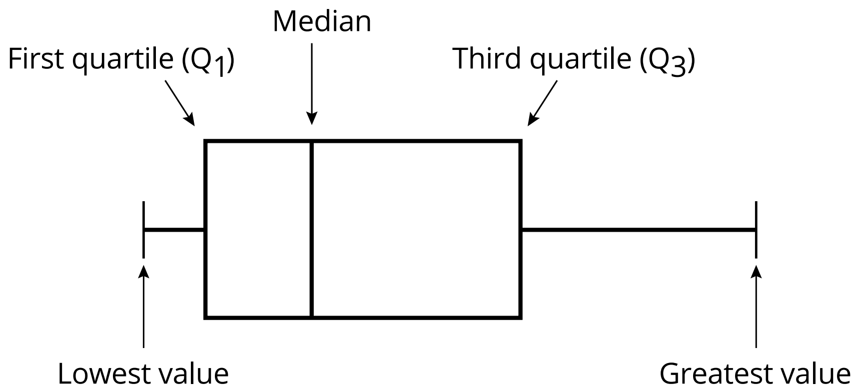
D1.2 Representation and Analysis of Data

represent and statistically analyse data from a real-life situation involving a single variable in various ways, including the use of quartile values and box plots

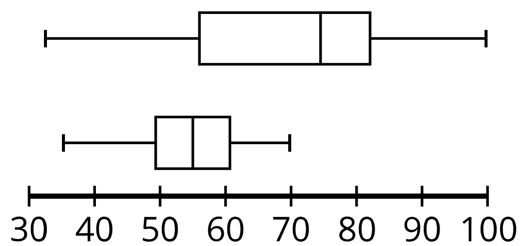
Teacher supports

Examples

- **real-life situations involving a single variable:**
  - lengths of commutes to school for a given group of students
  - amount of a given pesticide found in water samples collected from a local river
  - magnitudes of earthquakes in a given year, using the Richter scale
  - salaries of employees in an organization
  - amount of caffeine or sugar in various beverages
- **various representations:**
  - graphical:
    - box plot representing data involving a single variable:



- back-to-back box plots comparing the distributions of multiple groups:



- numerical:
  - measures of central tendency (mean, median, or mode, as appropriate for the data)
  - measures of spread (range and interquartile range)
  - five-number summary (lowest value, first quartile, median, third quartile, greatest value)
- **statistical analysis:**
  - descriptions of the centre, spread, outliers, and shape of the data set based on the numerical and graphical representations

Instructional Tips

Teachers can:

- support students in selecting an appropriate data set from a real-life situation involving a single variable;
- provide appropriate technological tools (e.g., statistical tools, spreadsheets, coding environments) as necessary for students to represent and analyse the data;

- revisit learning from earlier grades related to the graphical representations of data involving a single variable, such as histograms, stem-and-leaf plots, circle graphs, and various types of bar graphs, and distinguish between discrete and continuous data;
- support students in understanding the differences in the measures of central tendency and knowing when each one might be appropriate;
- continue to support students in developing their proportional reasoning skills, including the use of appropriate scaling in their representations;
- support students in expanding their communicative repertoire to include a broader range of related terminology and conventions, particularly for English language learners.

Teacher Prompts

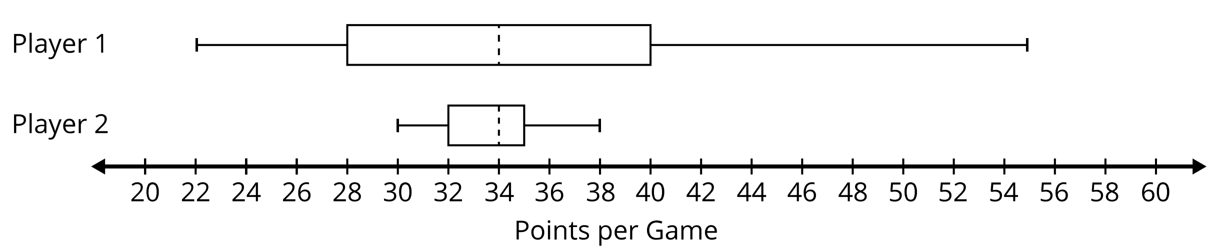
- How do you identify quartile values?
- What steps do you follow to create a box plot?
- When do you use a box plot to represent data?
- How do you know which data values might be outliers?
- What information does a box plot show that a histogram does not?
- What information does a stem-and-leaf plot show that a box plot does not?

Sample Tasks

1. Have students represent the CO_2 emissions in metric tons per capita from countries with populations of more than 20 million, using appropriate graphical and numerical representations.

2. Have students describe the shape, centre, and spread of the distribution of the number of cyclones formed over the Atlantic basin over the last 50 years, and any outliers.

3. Have students compare the distributions of the average number of points per game two basketball players scored in each season of their careers. They might create a box plot that looks something like the following:



4. Have students write code using subprograms to determine the range for a data set.

The following is an example of pseudocode for a subprogram that scans through a list of data to determine the minimum number.

findMinimum subprogram

subprogram **findMinimum** (**numList**)
numOfItems = number of items in the list
minimum = value of the first item in the list
itemNum = 2
repeat while (**itemNum**<=**numOfItems**)
if value of **itemNum** < **minimum**
minimum = value of **itemNum**
itemNum = **itemNum** + 1

The following is an example of pseudocode for a subprogram that scans through a list of data to determine the maximum number.

findMaximum subprogram

subprogram **findMaximum** (**numList**)
numOfItems = number of items in the list
minimum = value of the first item in the list
itemNum = 2
repeat while (**itemNum**<=**numOfItems**)
if value of **itemNum** < **maximum**
maximum = value of **itemNum**
itemNum = **itemNum** + 1

The following is an example of pseudocode that calls up the two subprograms to determine the range.

main program

range = 0.00
run subprogram **findMaximum**
run subprogram **findMinimum**
range = **maximum** - **minimum**
output “The range of the set of values is,” **range**

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.

D1.3 Representation and Analysis of Data

create a scatter plot to represent the relationship between two variables, determine the correlation between these variables by testing different regression models using technology, and use a model to make predictions when appropriate

Teacher supports

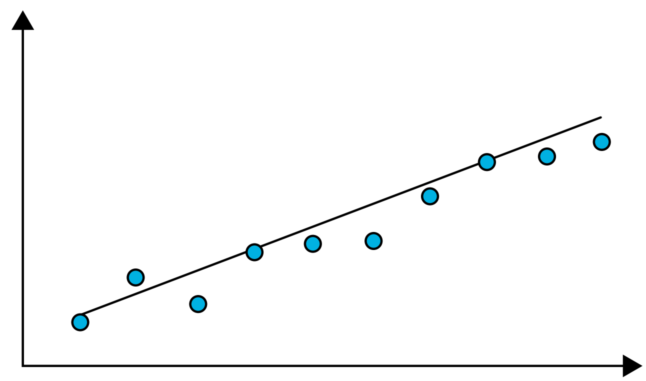
Examples

- **two variables with relationships:**
  - the fuel consumption of a car and its speed
  - the amount of saturated fats (in grams) and the number of calories in different granola bars
  - the amount of money borrowed and the interest rate that is offered
  - the size of the labour force and the employment rate
- **correlation:**
  - use of the correlation coefficient r to describe the strength and the direction of a linear relationship between two variables

Strength	Strong	Moderate	Weak	Weak	Moderate	Strong
$r =$	−1		0			1
Direction	Negative Correlation			Positive Correlation		

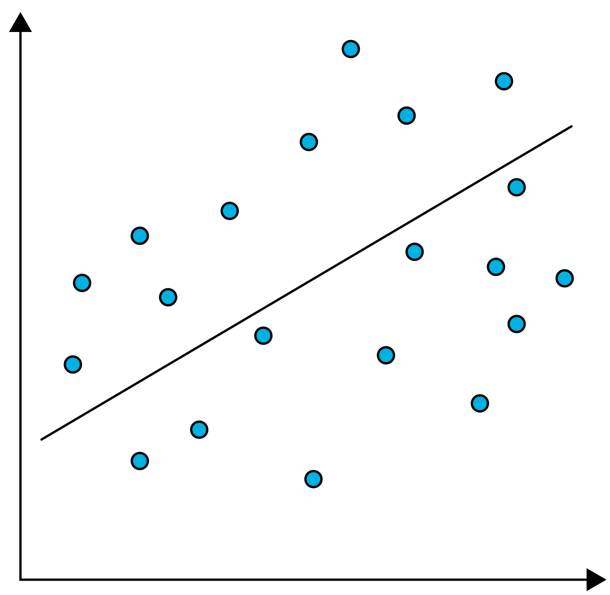
- strong positive linear correlation:

$r = +0.95$



- weak positive linear correlation:

$r = +0.3$



- **regression models constructed using technology:**
  - linear regression models
  - non-linear regression models

Instructional Tips

Teachers can:

- support students in describing the relationship observed on the scatter plot by discussing the direction (positive or negative), strength (strong, moderate, or weak), outliers, and form (linear or non-linear);
- ensure students have access to appropriate technological tools (e.g., statistical software, spreadsheets, coding environments when creating the scatter plot, determining the correlation, and testing different regression models;
- highlight, through the use of technology, linear and non-linear regression models as applications of linear and non-linear relations;
- support students in selecting the appropriate strategies to make predictions;
- facilitate conversations with students about when a regression model is and is not appropriate for making predictions;
- support students in expanding their communicative repertoire to include a broader range of related terminology and conventions, particularly for English language learners.

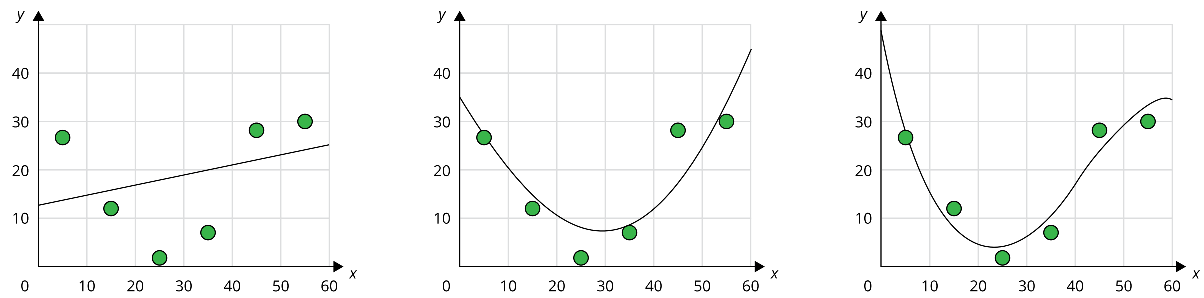
Teacher Prompts

- How do you make a scatter plot?
- What is the purpose of a scatter plot?
- In what ways can you describe the relationship between two variables on a scatter plot?
- What information does the correlation coefficient give us?
- How do outliers influence the value of the correlation coefficient?
- What is the difference between correlation and causation?
- What are the limitations involved in making predictions using regression models?

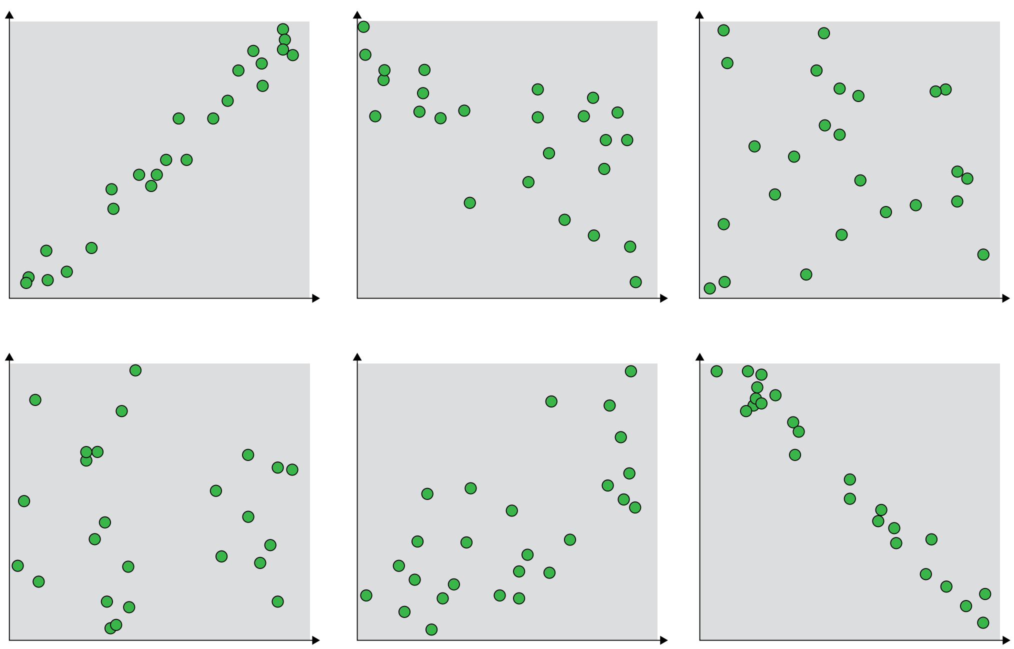
Sample Tasks

1. Have students create a scatter plot to show the relationship between average temperature and average wind speed at a given location over a period of time. Have students determine the appropriate regression model and use it to make predictions.

2. Give students different regression models of the same set of two-variable data and have them determine which model best represents the relationship.



3. Have students arrange six scatter plots of varying correlations according to the direction and strength of the association, and explain their reasoning.



Overall expectation

D2. Mathematical Modelling

apply the process of mathematical modelling, using data and mathematical concepts from other strands, to represent, analyse, make predictions, and provide insight into real-life situations

Specific expectations

By the end of this course, students will:

D2.1 Application of Mathematical Modelling

describe the value of mathematical modelling and how it is used in real life to inform decisions

Teacher supports

Examples

- value of mathematical modelling:
    - aids in developing an understanding of a real-life situation
    - aids in testing the impact of changes in a given real-life situation, such as ways to improve productivity and reduce associated costs
    - supports decision-making at all levels of society, such as:
        - guiding decisions that can help to improve sustainable economic development
        - guiding decisions that can help to reduce environmental degradation
- uses of the process of mathematical modelling in real life:
    - determining the efficacy of a vitamin supplement in improving health
    - predicting future sales based on the sales data from a given time frame
    - predicting weather patterns from past meteorological data tracked over time
    - predicting the amount of global carbon dioxide in future from past atmospheric data and core ice samples
    - predicting changes in the populations of endangered species in response to environmental changes
    - estimating the number of available seats on a train based on various assumptions, including anticipated cancellations and last-minute bookings

Instructional Tips

Teachers can:

- incorporate the process of mathematical modelling as students engage with real-life situations in the context of learning in other strands;
- support students in identifying the value of mathematical modelling in various real-life situations;

- introduce situations in the local community and ask students to reflect on potential related uses of mathematical modelling;
- support students in making the distinction between the use of a model to represent a mathematical concept and the process of mathematical modelling.

Teacher Prompts

- How can mathematical modelling be used to help us answer questions, in our own lives, in our communities, and in our society?
- What types of questions can mathematical modelling help us answer?
- How can mathematical modelling help us make predictions for the future?
- How can mathematical modelling help us identify where changes in policies and practices may be needed and how to achieve those changes?

Sample Tasks

1. Have students identify an event or situation in real life where mathematical modelling has been used. Have them discuss how mathematical modelling has helped inform decisions about that event or situation.

2. Have students place themselves in the role of a decision maker in their community or of the CEO of a company. Have them describe the types of decisions that can be made when applying the process of mathematical modelling.

3. Have students research careers that involve mathematical modelling.

D2.2 Process of Mathematical Modelling

identify a question of interest requiring the collection and analysis of data, and identify the information needed to answer the question

Teacher supports

Examples

- **questions of interest and information needed:**
  - Plastic waste contributes to the pollution of Earth's ecosystems. How does the proportion of plastic that ends up in recycling compare to the proportion that ends up in the garbage?
    - information needed may include:
      - the average amount of plastic waste that is discarded in a recycling bin
      - the average amount of plastic waste that is discarded in the garbage

            - the number of recycling bins distributed among the population of interest
    - Does listening to music while studying help students focus better?
        - information needed may include:
            - time spent listening to music while studying
            - scores on a memory test
    - Do younger teenagers need more sleep than older teenagers?
        - information needed may include:
            - amount of sleep that teenagers get at varying ages
            - measures of well-being (e.g., mood checks, energy levels, stress levels) from teenagers at varying ages after varying amount of sleep
    - What is the most appropriate location to place a wind turbine on a given site, based on the wind speeds at various locations on the site?
        - information needed may include:
            - wind speeds at various locations on the site
            - distance needed between a wind turbine and trees or other structures
    - Is there a relationship between the mass of a vehicle and its fuel efficiency or energy consumption?
        - information needed may include:
            - the mass of a vehicle
            - fuel efficiency or energy consumption of the vehicle
            - driving conditions (speed limits, weather conditions, the need to use climate control)

Instructional Tips

Teachers can:

- support students in selecting a question of interest to them and developing it in a way that can be answered by collecting and analysing information and data;
- support students in distinguishing between questions that require the process of mathematical modelling to answer them (often referred to as “messy questions” or “rich questions”) and questions that do not. For example, such questions may involve more than one variable or have different solutions depending on the assumptions made;
- facilitate conversations among students to discuss the information that is needed and the data that may need to be collected to build a mathematical model to answer their question of interest.

Note

Expectations D2.2 through D2.5 highlight the process of mathematical modelling and are therefore interconnected; they should be considered as a whole, and this should be reflected during instruction.

Teacher Prompts

- What is a question that interests you? How might mathematical modelling help you answer this question?
- What data/information would you need to answer the question?
- How could you collect or find this data?
- How would you develop a plan to analyse the (graphical/numerical) data?
- How do you identify what your assumptions are about your question?

Sample Tasks

Have students identify potential questions of interest, then have them identify the information required to build a mathematical model to answer the questions. Some sample topics for students to form questions of interest around could include the following:

- Have students discuss the information required to build a mathematical model to help plan school bus stops so that students do not have to walk too far to get to their nearest stop, yet the bus does not have to stop so often that a trip takes too long.
- In some places, drones are used to deliver parcels, rather than delivery trucks. In the first step of identifying a question of interest, have students discuss what sorts of situations might warrant the use of drones rather than delivery trucks and why.
- In contemporary society, people often rent rather than purchase entertainment, including using subscription services. For instance, most people have shifted from buying a copy of their favourite music album or movie to renting it for a limited time or streaming it. Have students discuss whether the choice to rent entertainment is based on cost, quality, convenience, environmental impact, or other factors, and determine what information they might need to gather to answer these questions.
- At some intersections, there are stop signs; others have traffic lights; and some have roundabouts. Have students determine what sort of information city planners might need to collect in order to determine what type of traffic-control device or measure is placed at each intersection. Have them discuss how their mathematical model might help them predict where they might change the placement of stop signs, traffic lights, and roundabouts in their neighbourhood, if they were the traffic designers.

D2.3 Process of Mathematical Modelling

create a plan to collect the necessary data on the question of interest from an appropriate source, identify assumptions, identify what may vary and what may remain the same in the situation, and then carry out the plan

Teacher supports

Examples

- **creating a plan to answer a question of interest:**
    - Plastic waste contributes to the pollution of Earth's ecosystems. How does the proportion of plastic that ends up in recycling compare to the proportion that ends up in the garbage?
        - data that may need to be collected:
            - data from a survey of the local community, school, classroom, city, town, or municipality to find out the amount of plastic that is discarded in recycling and the amount of plastic that is discarded in the garbage for a specific time frame or for the different seasons of the year
            - data on the number of recycling bins distributed among the population of interest
        - possible assumptions:
            - not all of the plastic waste is recyclable, but everything that is recyclable is being recycled
        - what may vary:
            - the amount of plastic waste at different times of the year
        - what may remain the same:
            - the capacity of the recycling bins and garbage bins
    - Is there a relationship between the mass of a vehicle and its fuel efficiency or energy consumption?
        - data that may need to be collected:
            - data on the mass of specific vehicles and their fuel efficiency or energy consumption
        - possible assumptions:
            - the data published in the secondary source is accurate
            - the fuel efficiency or energy consumption for each vehicle is measured using the same type of measuring tools and calculated using the same methods
        - what may vary:
            - mass of vehicles
            - fuel efficiency or energy consumption rates
            - types of vehicles
        - what may remain the same:

- the fuel efficiency or energy consumption rate is measured on a similar driving conditions (e.g., speed limits, weather, the need to use climate control)

Instructional Tips

Teachers can:

- support students in selecting the most appropriate methods of data collection in order to collect the information needed to answer their question (e.g., surveying a sample of individuals, conducting an experiment, testing a design prototype, data collected from the census);
- provide students with access to a variety of print and digital resources to research and collect data;
- facilitate conversations among students to enable them to discuss and reflect on the assumptions made, what can vary, what can remain the same, and how these elements can affect their plan to answer their question of interest;
- facilitate discussions about the elements of and challenges in developing and carrying out their plan.

Note

Expectations D2.2 through D2.5 highlight the process of mathematical modelling and are therefore interconnected; they should be considered as a whole, and this should be reflected during instruction.

Teacher Prompts

- What is your plan to collect the data/information?
- What are the steps needed in your plan?
- What are the sources of your data?
- What assumptions are you making in choosing which data to collect?
- Which aspects of the situation change? Which remain the same?
- What potential biases might be involved in the data collection?

Sample Tasks

1. Have students map out all the steps of their plan. Have students determine what information they need for their question of interest.
    - For instance, if they are doing the modelling task of planning school bus stops, they might need to look at the school bus routes to see how far apart the stops are currently.

- Students might want to examine why stops are closer together in some areas. They might also look at how long it takes a bus to start and stop, collect data as to how far students are willing to walk to a bus stop, and so on.

2. Have students identify possible assumptions; for example, who gets picked up by the bus first and dropped off last, or the importance of shorter wait times at certain times of the day.

3. Have students identify what they think is important to consider. For example, they may want to determine the cost of constructing each bus stop and other considerations for working within a fixed budget.

D2.4 Process of Mathematical Modelling

determine ways to display and analyse the data in order to create a mathematical model to answer the original question of interest, taking into account the nature of the data, the context, and the assumptions made

Teacher supports

Examples

- **displaying and analysing the data in order to create a mathematical model:**
  - Plastic waste contributes to the pollution of Earth's ecosystems. How does the proportion of plastic that ends up in recycling compare to the proportion that ends up in the garbage?
    - creating the mathematical model may involve calculating the average mass of the plastic waste in a recycling bin and a garbage bin and determining its percentage relative to the mass of other recyclables and other garbage in the same bin for different locations and for different times; it may also involve using this average to calculate the total amount of plastic in recycling and in the garbage for the entire population of interest
    - displaying the data may involve representing it as a proportion of plastic in recycling to plastic in the garbage using a circle graph, or as a multiple bar graph to show a comparison of various locations or different times or seasons
    - analysing the data may involve comparing the percentages and the actual amounts of plastic being recycled and being placed in the garbage at various locations or at different times or seasons
  - Is there a relationship between the mass of a vehicle and its fuel efficiency or energy consumption?
    - displaying data collected on the mass of a vehicle and its fuel efficiency or energy consumption may involve using a scatter plot
    - analysing the data may include:
      - describing the relationship between the mass of a vehicle and its fuel efficiency or energy consumption by examining the scatter plot and

identifying the direction (positive or negative), strength (strong, moderate, or weak), and form (linear or non-linear) of the relationship, and any outliers

- using technology to create an appropriate linear (or non-linear) regression model of the situation; for example, $e = 0.004 \times m + 2.015$, where e represents the fuel efficiency in litres per 100 kilometres, and m represents the mass of the vehicle in kilograms

Instructional Tips

Teachers can:

- support students in recognizing that the models they create are dependent on the assumptions they are making;
- support students in selecting the most appropriate way to display and analyse their data, based on the type of data they gathered (e.g., a box plot for data involving a single variable, a scatter plot for data involving two variables);
- focus students' attention on the fact that they may need to create different types of mathematical models, including a visual representation or diagram, a table, a graph, a formula, and/or an equation, to answer their question of interest;
- engage students in discussions about the features, strengths, and limitations of various mathematical models;
- ensure that students have access to appropriate technological tools (e.g., statistical tools, spreadsheets, coding environments) when creating mathematical models.

Note

Expectations D2.2 through D2.5 highlight the process of mathematical modelling and are therefore interconnected; they should be considered as a whole, and this should be reflected during instruction.

Teacher Prompts

- What mathematical information and skills might be needed in order to build the model?
- What representations, tools, technologies, and strategies will you use to build your model?
- What are the most appropriate ways to display your data? Discuss the different options available and which one or more of these options best suits the data, based on the nature of the data.
- Does the visual representation accurately model the data? How do you know?

Sample Tasks

Have students brainstorm, in small groups, how they want to display the data they have gathered. For example, they may use a spreadsheet to show the costs associated with having more bus stops, create graphs that show the different amounts of time that trips take based on the number of times the bus has to stop, generate a graphic showing the spread of distances that students are comfortable walking, or create a city/town map showing where they would place the bus stops.

D2.5 Process of Mathematical Modelling

report how the model can be used to answer the question of interest, how well the model fits the context, potential limitations of the model, and what predictions can be made based on the model

Teacher supports

Examples

- **assessing the model and using it to make predictions:**
    - Plastic waste contributes to the pollution of Earth's ecosystems. How does the proportion of plastic that ends up in recycling compare to the proportion that ends up in the garbage?
        - assessing the model:
            - Does your mathematical model describe the amount of plastic waste in recycling and the amount of plastic waste in the garbage as proportions?
            - Does your mathematical model account for your assumption that any plastic that is recyclable is in the recycling bin?
            - How does your mathematical model use the fact that all the recycling bins for your population of interest are the same size, and all the garbage bins are the same size?
            - How does your mathematical model account for the question of whether the amount of plastic being recycled or disposed varies at different times of the year?
        - making predictions:
            - What percentage of plastic waste is recycled for your population of interest? Does this answer make sense? Why or why not?
            - If 3000 tonnes of plastic is recycled in a given time period, how much plastic ends up in the garbage in that same time period?
    - Is there a relationship between the mass of a vehicle and its fuel efficiency or energy consumption?
        - assessing the model:
            - Does this model show a strong correlation on the scatter plot between the mass of a vehicle and its fuel efficiency or energy consumption?
            - Are there any outliers on the scatter plot that could affect the correlation?
        - making predictions:
            - What is the predicted value of the fuel efficiency or energy consumption of a vehicle with a given mass within and outside the range of data points?

Instructional Tips

Teachers can:

- facilitate conversations with students to discuss how different models can be created to answer their question of interest based on their assumptions;
- engage students in discussions about the features, strengths, and limitations of various mathematical models;
- support students in reflecting on the reasonableness of the model they have created to answer their question and whether they need to revise the model to better reflect the assumptions they have made;
- draw students' attention to the limitations of a model in predicting values beyond the range of the data collected, as well as limitations due to any biases in the data collection;
- support students in creating a report (e.g., infographic, presentation, another format of the student's choosing) that provides all the necessary information to answer their question of interest, including their mathematical models, ensuring that enough detail is given to inform possible decisions.

Note

Expectations D2.2 through D2.5 highlight the process of mathematical modelling and are therefore interconnected; they should be considered as a whole, and this should be reflected during instruction.

Teacher Prompts

- Does your model help you to answer your question? Did you need to revise the model? Why?
- Does your model allow you to make predictions?
- What predictions can be made based on the model?
- What are the limits of the model?

Sample Tasks

Have students share the strengths and limitations of their model and explain how the model helped them answer their question and make predictions. For instance, they might present their results as a report or presentation to convince a school board or school bus company that their plan addresses a bus-stop problem in the most efficient and economical way and include their suggestions for the placement of bus stops using the data displays that they created in D2.4. Have other students in the class act as the school board or bus company officials to ask questions about the model to see if the model needs to be refined.

E. Geometry and Measurement

Overall Expectation

By the end of this course, students will:

E1. Geometric and Measurement Relationships: demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations

Overall expectation

E1. Geometric and Measurement Relationships

demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations

Specific expectations

By the end of this course, students will:

E1.1 Geometric and Measurement Relationships

research a geometric concept or a measurement system to tell a story about its development and use in a specific culture or community, and describe its relevance in connection to careers and to other disciplines

Teacher supports

Examples

- **stories that students may share involving a geometric concept or a measurement system:**
  - The relationship between the side lengths of a right triangle has been attributed to Pythagoras (c. 570–490 BCE). However, it was known about by people in other cultures before this, such as the ancient Egyptians, who were using it around 1000 years before Pythagoras. They used a rope with 12 knots, evenly spaced, to form a 3-4-5 triangle with a 90° angle to help build the walls of the pyramids. This 3-4-5 relationship is still used today in construction to determine whether a corner is “square” (90°) or not.
  - For centuries, many cultures have used body parts to measure length. These ancient measurements were based on what people could relate to as a unit of measure. For example, a hand was used to measure the height of a horse, and, at some point over the years, since the size of hands differs, horse traders agreed that a “hand” would be

equivalent to 4 inches (10.16 cm). Today a horse’s height is still referred to in terms of hands.

Instructional Tips

Teachers can:

- on an ongoing basis, both formally and informally, encourage students to bring into the classroom real-life stories they have gathered about mathematical concepts, in order to enhance their understanding of these concepts and make connections between them;
- build an authentic and inclusive learning environment where students are encouraged to learn about the diversity of knowledge systems from around the world, including Indigenous ways of knowing.

Note

Students can seek out real-life stories through conversations with people in their families or communities, or through print and digital resources. They may need guidance on seeking information about new perspectives on mathematics. The aspect of student choice in this expectation may also involve the teacher taking the stance of a co-learner as they support students in exploring stories of various cultures.

Teacher Prompts

- Why is the geometric concept or measurement system you chose of interest to you or relevant to you?
- What do you find most interesting about your story?
- What challenges did you face while researching this concept or system?
- Describe some ways you can connect the concept or system you researched to your own learning, careers that you know about, or your daily life.
- What do you think connects the different stories you heard in class and the mathematical concepts you have encountered?
- What are some similarities and differences between your story and concepts or systems researched by your peers?
- Describe any connections you notice between the geometric concept or measurement system you chose and the natural world.

Sample Tasks

4. Have students brainstorm possible geometric concepts or measurement systems that they could research. Ask them to choose a concept or system of interest from the list and gather information about its socio-historic development in a culture of their choice. Also ask them to describe how their concept or system is used today and identify its connections to careers and other disciplines. Once students have gathered their information, have them decide how they would like to tell the story of the development of the concept or system to the class.

5. Have students collaborate to create a conversion chart showing the relationship between the different units of measurement that they have investigated.

6. Have students collaborate to create a collection of images that demonstrate and tell the story of different ways that geometric concepts have been used across many different cultures over time.

E1.2 Geometric and Measurement Relationships

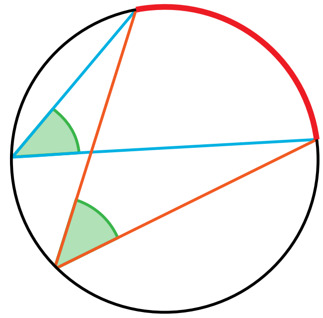
create and analyse designs involving geometric relationships and circle and triangle properties, using various tools

Teacher supports

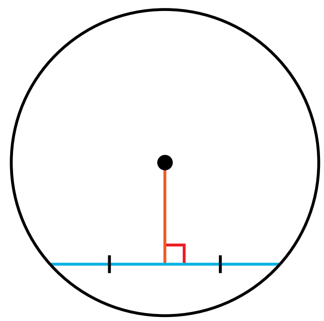
Examples

- **geometric relationships involving:**
    - parallel and intersecting lines:
        - corresponding angles
        - alternate angles
        - co-interior angles
        - opposite angles
        - supplementary angles
        - complementary angles
    - polygons:
        - sum of the interior angles
        - sum of the exterior angles

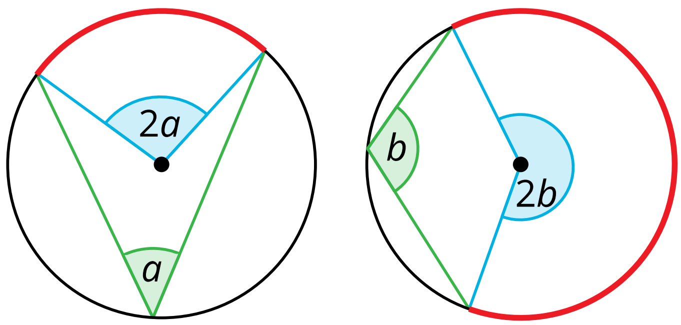
- **circle properties may include:**
  - inscribed angles subtended by the same arc are congruent:



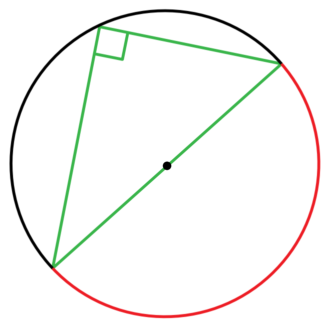
  - a perpendicular line from the centre of a circle to a chord bisects the chord:



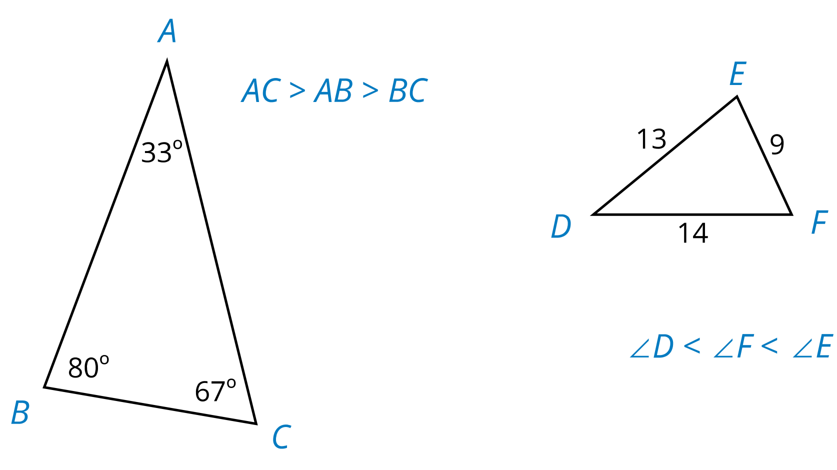
  - the measure of the central angle is always twice the measure of an inscribed angle subtended from the same chord:



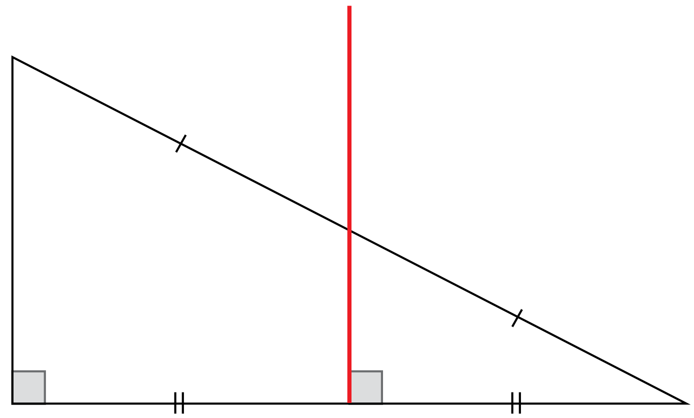
- an angle subtended from the diameter is always a right angle:



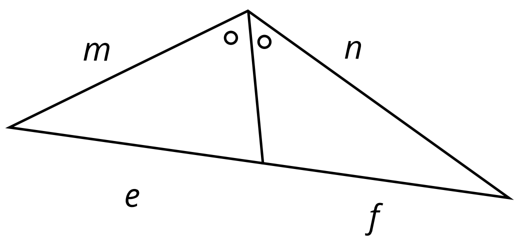
- **triangle properties may include:**
  - the combined length of any two sides of a triangle is always greater than the length of the third side
  - the interior angles of a plane triangle always add up (sum) to 180° (e.g., 70° + 60° + 50° = 180°)
  - the exterior angles of a plane triangle always add up (sum) to 360° (e.g., 110° + 120° + 130° = 360°)
  - the interior angle of a plane triangle and its side-extension exterior angle always add up (sum) to 180° (e.g., 130° + 50° = 180°)
  - the longest side and the largest angle of a triangle are opposite one another; the shortest side and smallest angle are opposite one another; and the mid-sized side and mid-sized angle are opposite one another:



- the perpendicular bisector of either of the two smaller sides of a right triangle bisects the hypotenuse of that triangle:

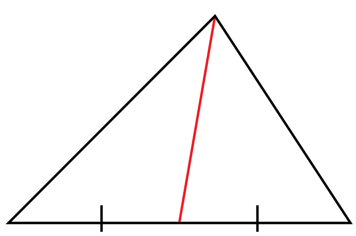


- the ratio of the segments created by an angle bisector is equivalent to the ratio of the sides adjacent to those segments:



$$\frac{m}{n} = \frac{e}{f}$$

- the median of a triangle divides the triangle into two equal areas:



- **tools for creating and analysing designs:**
  - paper and pencil
  - string
  - beads
  - square tiles
  - pattern blocks

- geoboards
- geometric drawing devices such as compasses, protractors, and rulers
- dynamic, interactive geometry tools
- coding programs

Instructional Tips

Teachers can:

- provide tasks with various criteria, constraints, and parameters so that students have the opportunity to create different geometric designs and make connections to various geometric concepts;
- reinforce learning about geometric relationships from elementary mathematics, including properties of parallel and intersecting lines and of interior and exterior angles of polygons;
- support students in using technology to explore circle and triangle properties and to reinforce learning from the elementary grades about geometric properties;
- highlight careers that use geometry and/or measurement, such as architect, skilled artisan, and tool and die maker.

Teacher Prompts

Geometric relationships:

- How can you predict the size of an inscribed angle of a circle if you are given the corresponding central angle of the circle?
- How can you predict the size of a central angle of a circle if you are given the corresponding inscribed angle?
- What is the relationship between the sum of the measures of the interior angles of a polygon and the number of sides of the polygon?
- How can you determine the exterior angle measure of a regular polygon with *n* sides?

Geometric designs:

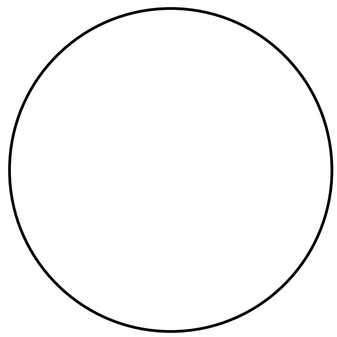
- What geometric relationships do you notice in this geometric design?
- What geometric relationships did you use to make your design?

Sample Tasks

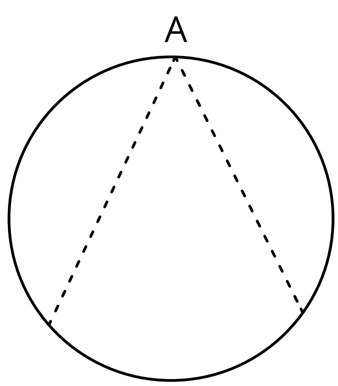
1. Have students construct various sizes of triangles and measure their side lengths and interior angles. Ask them what they notice about the angles of a triangle and the lengths of the sides opposite those angles.

2. Have students explore circle properties using paper-folding activities. Have them first fold paper to locate the centre of a circle and then continue folding to determine various circle properties. The following is an example of a task that involves folding paper to determine the relationship between an inscribed angle of a circle and its corresponding central angle:

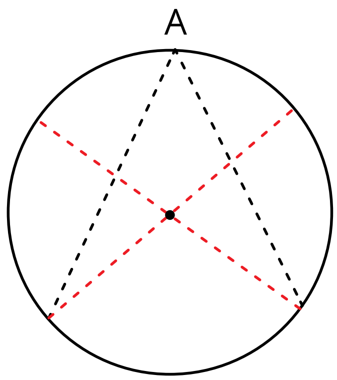
   - Cut out a circle:



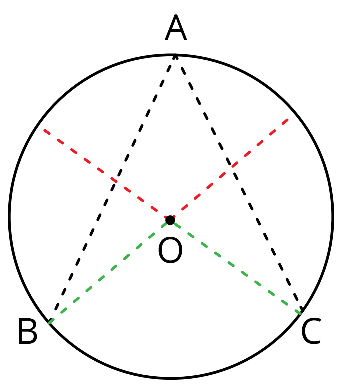
   - Fold two chords on the circle to create the inscribed angle A:



- Fold along two diameters, each of which shares an endpoint with one of the chords, to find the centre of the circle, as shown below:



- Measure angle BAC (the inscribed angle) and angle BOC (the central angle), and state what you notice about their angle measures:



3. Provide students with a set of instructions that enable them to demonstrate their understanding of a circle property. For example:

   - Step 1. For any circle, draw a chord.
   - Step 2. Draw a segment from the midpoint of the chord to the centre of the circle.
   - Step 3. Measure the angles formed by the segment and the chord.
   - Step 4. Repeat with three different chords.

   What appears to be true about the angles that are formed each time?

4. Have students fold paper to create geometric designs involving:

   - finding the centre of a circle
   - creating an equilateral triangle from a circle
   - creating a hexagon from a rectangle and from a circle.

5. Have students share a geometric design that is of interest to them and identify possible geometric relationships within the design.

6. Have students create a geometric design that includes given geometric properties.

7. Have students create a scale diagram of the playing surface of a sport of their choice. Have them share the significance to the sport of the geometric design on the playing surface.

E1.3 Geometric and Measurement Relationships

solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities, using various representations and technology, when appropriate

Teacher supports

Examples

- **units from various measurement systems**
    - units of length – arm lengths, handspans, inches, feet, yards, miles, millimetres, centimetres, metres, kilometres, light years, stride lengths
    - units of area – square feet, square yards, square centimetres, square metres, acres, hectares
    - units of capacity – drops, scoops, handfuls, bagfuls, pints, cups, gallons, quarts, bushels, millilitres, litres
    - units of volume – cubic feet, cubic yards, cubic millimetres, cubic centimetres, cubic metres
    - units of mass – grams, kilograms, tonnes
    - units of digital storage – gigabytes, megabytes, terabytes
    - units of time – nanoseconds, seconds, minutes, days, years, centuries, phases of the Moon, seasons
- **representations and technology:**
    - various measuring tools, such as grid paper, rulers, and base 10 blocks
    - open double number lines
    - dynamic, interactive geometry tools
    - coding

Instructional Tips

Teachers can:

- facilitate class discussions to engage students in sharing different ways of measuring that they have encountered in real life or in their other courses;
- support students in posing and solving problems that are relevant to them;
- respectfully share ways of measuring from various cultures and communities;
- support students in making connections among units in the context of the problems they are solving;
- support students in choosing tools and representations that are appropriate for the context;
- provide tasks that support students in developing proportional and spatial reasoning;
- support students in identifying the precision to which measurements are needed based on the context of the problem;
- have students alter or write code to solve problems, such as converting between units or between measurement systems.

Note

The focus of the learning in this expectation is for students to be able to solve authentic problems that may involve different types of units.

Teacher Prompts

- Which unit is appropriate for this context?
- In the Olympics, some sports are timed to the hundredth of a second, and some are measured to the thousandth of a second. Which sporting events require more precise times? Why?
- What is an example of a system of measurement other than metric or imperial?
- If the unit of measure changes from centimetres to millimetres, are more or fewer units needed to measure the same distance? Explain.
- What measurements are usually measured in imperial, and what measurements are usually measured in metric?
- What measurements are regularly measured in both imperial and metric?

Sample Tasks

1. Provide students with different scenarios of how cultures or communities use various measurement systems, and ask them to solve related problems. For example, in the Islamic tradition, a donation of charity or alms on Eid is called a Sāʿ. Traditionally, this is the volume of grain that can be gathered in your two hands held together, four times. Have students estimate how much grain this is in grams.

2. Ask students to solve problems involving time. For example:

- Is a million seconds almost as many as a billion seconds, or not? Which is closer to the number of seconds in a month?
- How much greater is a millisecond than a nanosecond? Could you tell the difference if something took a millisecond instead of a nanosecond?

3. Ask students to share a recipe that is relevant to them, their family, and/or their community. Have them exchange recipes and pose questions for other students to answer, such as:

   - What ingredient do you need the most of, and how can you tell?
   - If you are missing one of the measuring tools, how can you use another measuring tool to measure the appropriate amount?
   - If the recipe uses mass, how can you convert it to use capacity measuring tools?

4. Have students estimate the distance from one location to another, which may include using a personal referent such as stride length. Next, ask them to measure the distance using both metric and imperial units. Have them discuss what they notice about their estimate and the different measurements.

5. Have students solve problems that involve different units of measure from the same measurement system. For example:

   - What is the speed per second of downloading digital information on the computer and Internet connection you are using? How much digital information can you download in 1 minute?
   - How many millilitres of liquid can you pour into a container with a capacity of 1.5 L?
   - A circle has an area of 124.5 mm^2. What is its diameter in centimetres?

6. Have students solve problems that involve comparing measures from different measurement systems. For example:

   - Which rectangular community garden has the least area?
     Garden A: 12.5 m × 5.8 m
     Garden B: 12.5 feet × 5.8 feet
     Explain why.
   - How much greater is 12 metres than 12 feet?
   - Which is the better fuel consumption rating: 7.5 litres per 100 kilometres or 52 miles per gallon? Explain why.
   - On Earth, how much does a handful of rocks weigh in grams? in pounds?

7. Have students solve problems involving scale diagrams, such as using a map to determine the distance travelled or a blueprint to determine the amount of material needed.

8. Have students solve problems that involve using unconventional units to measure. For example: How many of the same type of coin are needed to go around the circumference of Earth?

E1.4 Geometric and Measurement Relationships

show how changing one or more dimensions of a two-dimensional shape and a three-dimensional object affects perimeter/circumference, area, surface area, and volume, using technology when appropriate

Teacher supports

Examples

- **dimensions that may change:**
  - side length(s) of a polygon
  - radius or diameter of a circle
  - side length(s) and/or height of a prism or pyramid
  - height and/or radius of a cylinder or cone
- **technology:**
  - coding
  - interactive simulations
  - spreadsheets
  - dynamic, interactive geometry tools

Instructional Tips

Teachers can:

- support students in visualizing, verbalizing, and verifying problems by having them:
  - visualize the results of a change in one or two dimensions of a two-dimensional shape or one, two, or three dimensions of a three-dimensional object;
  - verbalize to other students what they were visualizing in their thinking;
  - verify their thinking with or without the use of technology;
- provide tasks that involve making connections to proportional reasoning, such as doubling or tripling side lengths.

Teacher Prompts

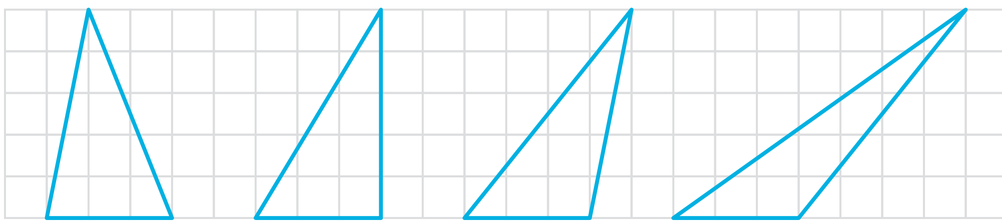
- If you double the height and the base of a rectangle, what happens to its perimeter? What happens to its area?
- If one of the dimensions of a right prism increases, what needs to happen to the other dimensions for the volume to stay the same?
- If the height of a cylinder is halved, and all other dimensions stay the same, what happens to the area of its curved surface?
- Is it possible to have pyramids with different dimensions and the same volume? Explain why or why not.

- Is it possible to have right prisms with different dimensions and the same surface area? Explain why or why not.
- If a cylinder and a cone have the same base and height, how much taller must the cone be in order to have the same volume as the cylinder?

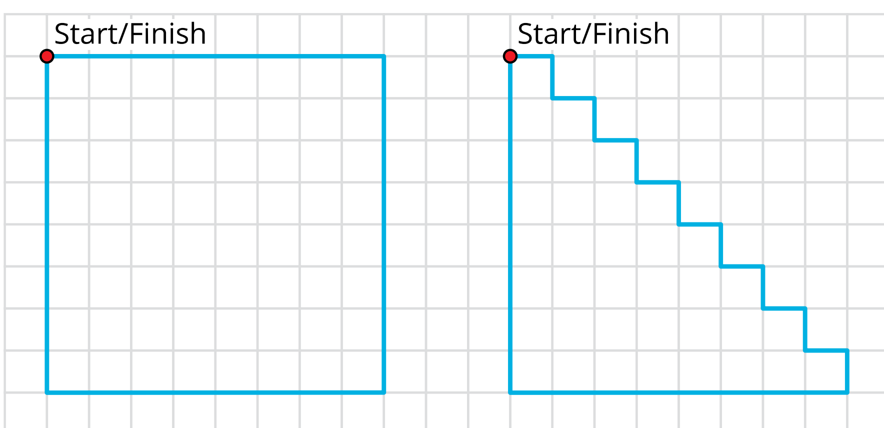
Sample Tasks

1. Provide students with questions that require them to understand which dimensions of a two-dimensional shape need to change in order to change its perimeter or its area. For example:

   - Which triangle in the image below has the greatest perimeter? Justify your choice.
   - Which triangle in the image below has the greatest area? Justify your choice.



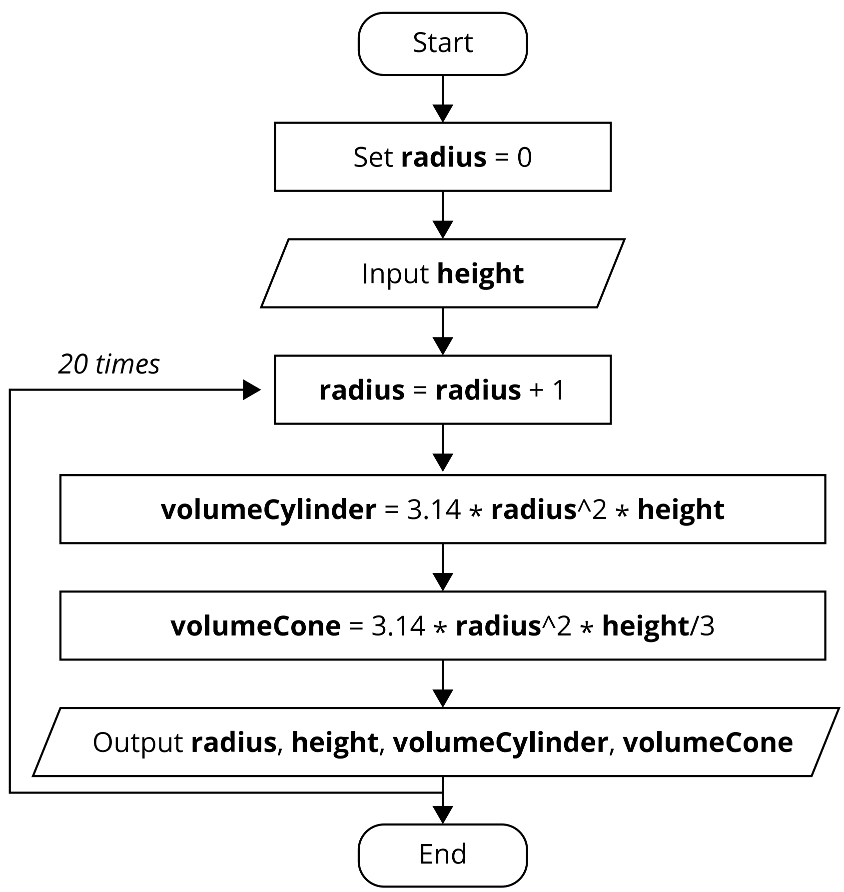
   - Two runners follow different paths along the streets in their neighbourhood. If they start and finish at the same spot, as shown in the image below, who will run farther? Justify your choice.



2. Have students predict whether the circumference or the area of a circle will change more if the diameter increases. Then have them use various measures for the circumference to verify or refute their prediction.

3. Have students use interlocking cubes to make a rectangular prism and then determine its volume and surface area. Next, have them make another rectangular prism with double the length, keeping the other measurements the same, and then determine its surface area and volume. Have them compare the volumes and surface areas of the two prisms, and ask what they notice. Have them predict what will happen to the volume and surface area if the length of the original prism is tripled and the other measurements remain the same. Then, have them verify or refute their prediction. Repeat for doubling and tripling the original prism's length and width. Repeat for doubling and tripling the original prism's length, width, and height.

4. Provide students with the following flow chart. Ask them to read the flow chart and explain what it is modelling. Have them write the code and describe how the volumes of the cylinders and their corresponding cones are affected when the radius increases by 1 unit each time. Students could then add code to plot the values in order to see the changes graphically.



5. Have students alter the code they created in the previous task or use a spreadsheet to compare the volumes and surface areas of prisms and pyramids when there is a change in one dimension, two dimensions, and three dimensions.

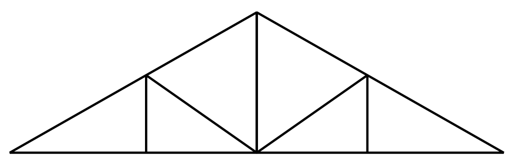
E1.5 Geometric and Measurement Relationships

solve problems involving the side-length relationship for right triangles in real-life situations, including problems that involve composite shapes

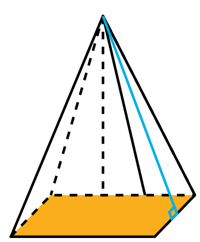
Teacher supports

Examples

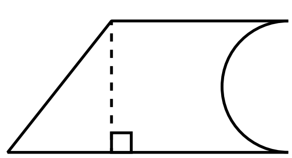
- **real-life situations involving right triangles:**
  - creating or analysing a design for an outdoor space
  - determining whether a wall is straight
  - determining whether a box is square (has 90° angles)
  - building a ramp to improve accessibility
  - building a garden bed
  - building a bridge
  - creating plans for a dwelling
  - building a roof truss:

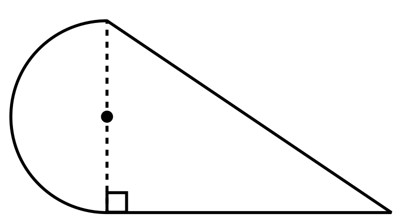


  - designing a pyramid-shaped structure such as a tent:



- **composite shapes involving right triangles:**





Instructional Tips

Teachers can:

- support students in posing and solving problems that are relevant to them;
- use geometric representations to reinforce students' understanding of the side-length relationship for right triangles; that is, the area of the square extending from the hypotenuse of a triangle is equal to the sum of the areas of the squares extending from the other two sides;
- support students in identifying the precision to which measurements are needed based on the context of the problem.

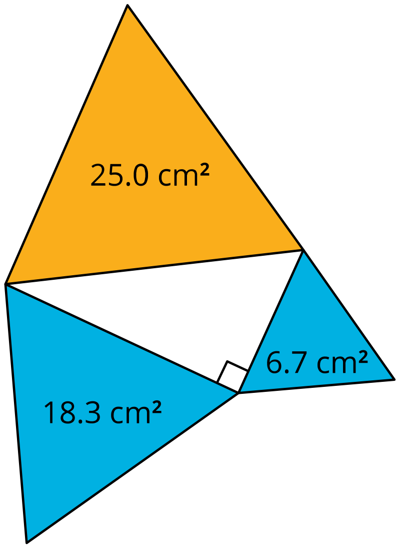
Teacher Prompts

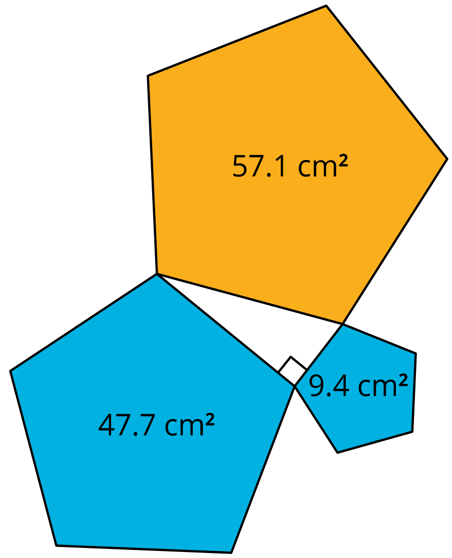
- Identify a real-life situation that might require knowing the relationship between the lengths of the sides of a right triangle.
- Are all triangles that have side lengths in the ratio of 3 to 4 to 5 right triangles?
- What is the relationship between the side lengths of any right triangle?
- If the three side lengths of a right triangle are 5 units, 12 units, and 13 units, name the side lengths of a triangle similar to it.
- The net of a square-based pyramid is a composite shape. What are the basic shapes in the net? What dimensions are needed to find the slant height of one of the sides of such a pyramid?

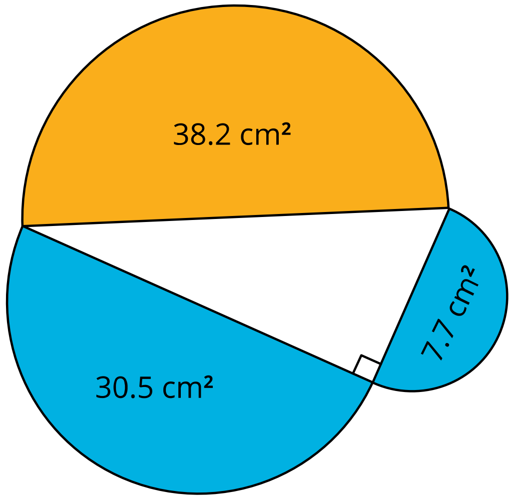
Sample Tasks

1. Have students take a photo of a right triangle in a real-life context. Have them determine the length of its hypotenuse using the side-length relationship. Have them verify their result by measuring the hypotenuse using a measuring tool. If the real-life right triangle can be measured, have students compare its measures with those of the photo to determine its scale factor.

2. Have students use technology to verify the relationship of the sides of a right triangle using shapes other than a square, such as in the examples shown below.

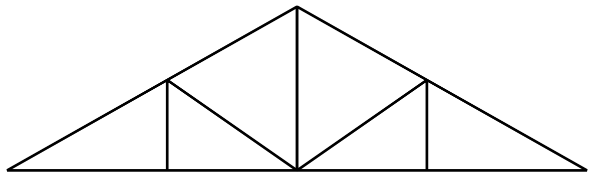




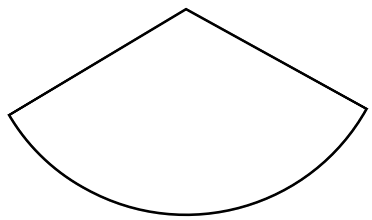


3. Have students create a right triangle with side lengths of 5 cm, 12 cm, and 13 cm. Have them create three triangles that are similar to this triangle. Ask them to verify that the relationship between the sides holds for these similar triangles.

4. Have students determine how much wood is needed to construct a roof truss that looks like the image below and has a pitch of 5:12.



5. Have students cut and fold a sector of a paper circle to form a cone, as shown below.



   Ask them to identify and measure the height of the cone and the radius of the circle that forms the opening of the cone. Ask them to visualize the right triangle that is formed between the height, the radius, and the slant height of the cone. Have them use the relationship of the sides of a right triangle to determine the slant height of the cone. Then ask them to measure the slant height directly using a ruler and then compare their results.

6. Have students create a composite shape that includes a right triangle, and ask them to determine its perimeter and area.

7. Have students work in small groups to design the putting green for one hole of a mini-golf course. Have them create a scale drawing of their design that includes at least one right triangle. Have them identify the key measurements needed to create their putting green and its hole and determine the scale relative to the actual measurements. Ask them to determine the total amount of surface area required for the putting green. Then have them construct the putting green using cardboard and masking tape. Once students have constructed their putting greens, set up a course outside and have students play a round of mini-golf, where they can practise their integer skills while keeping score.

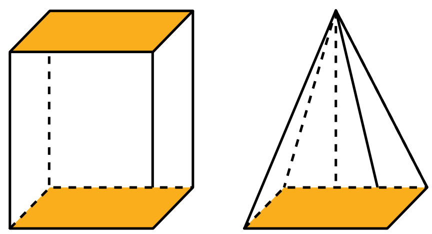
E1.6 Geometric and Measurement Relationships

solve problems using the relationships between the volume of prisms and pyramids and between the volume of cylinders and cones, involving various units of measure

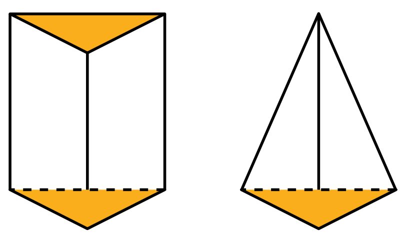
Teacher supports

Examples

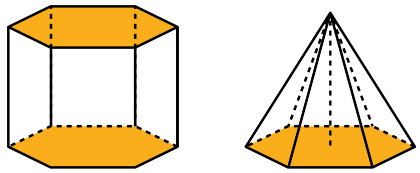
- **prisms and pyramids with the same height and base area:**
  - rectangle-based prism and pyramid:



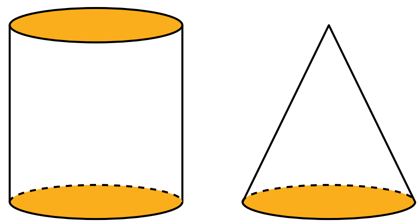
  - triangle-based prism and pyramid:



  - hexagon-based prism and pyramid:



- **cylinder and cone with the same height and base area:**



Instructional Tips

Teachers can:

- create tasks that enable students to make sense of the relationships between prisms and pyramids, and between cylinders and cones, through manipulating solids and pouring materials;
- support students in generalizing the relationships by having them experiment with a variety of base shapes, including regular bases, irregular bases, and objects with different-shaped bases, such as a heart-shaped box;
- support students in identifying the precision to which measurements are needed based on the context of the problem;
- use appropriate representations and materials to reinforce students' understanding that the relationship stays the same whether it is capacity (mL) or volume (cm^3) that is being measured.

Note

The volume of a pyramid is one third the volume of a prism with the same base and height. The same relationship holds for cones and cylinders.

Teacher Prompts

- If you know the volume of a prism, how can you determine the volume of a pyramid with the same height and base as the prism?
- If you know the volume of a cone, how can you determine the volume of a cylinder with the same height and base as the cone?

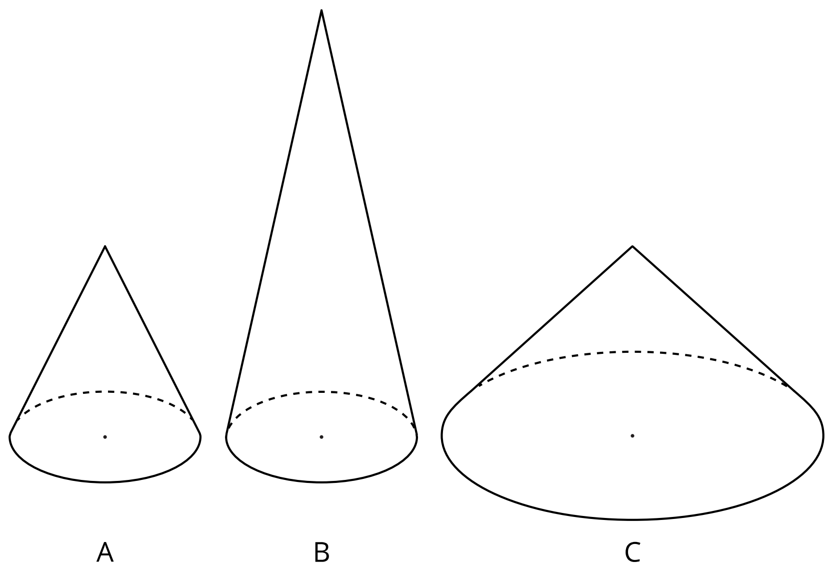
Sample Tasks

1. Provide students with several different sizes of prism and pyramid containers with the same height and base. Also provide cylinder and cone containers with the same height and radius. Provide students with a filling substance such as water or sand, and ask them to determine how many

pyramids full of the substance it takes to fill the corresponding prism. Have them do the same with the cones and cylinders.

2. Ask students which of the following cones has the greatest volume and to justify their choice.

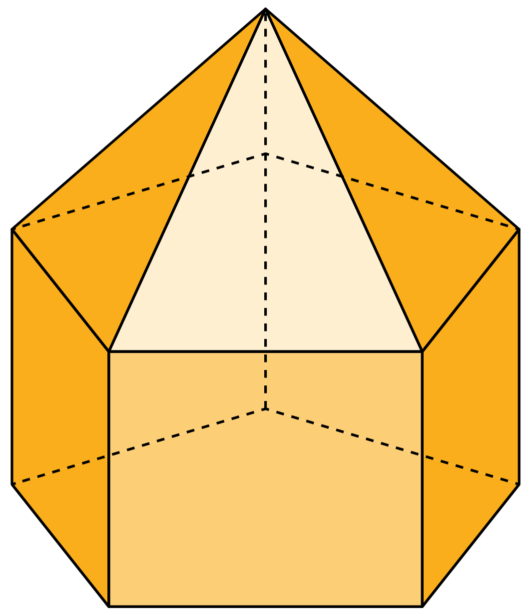
   - Cone A: Height is equal to the diameter of its circular base.
   - Cone B: Height is double the height of cone A and diameter is the same as cone A.
   - Cone C: Height is equal to that of cone A and diameter is twice that of cone A.



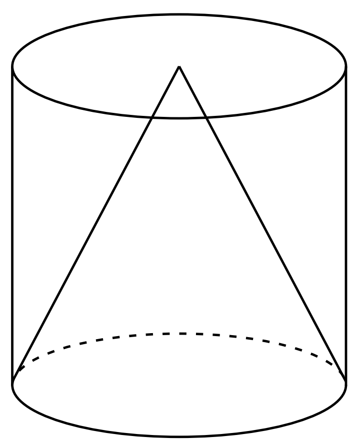
3. Show students an image of a large real-life object that closely resembles a pyramid or a cone, and ask them to estimate its volume.

4. Have students solve problems that involve the volume of composite figures and can be solved using the relationship between the volume of a pyramid and the volume of a prism or the relationship between the volume of a cone and the volume of a cylinder. For example:

   - The container shown below is made up of a pentagonal prism and a pentagonal pyramid that both have a height of 10 cm and a base area of 30 cm^2. What is the total volume of the container? What are the side lengths of the regular pentagonal base of the prism?



- What is the volume of the space not occupied by the cone inside the cylinder shown below, if both shapes have a height of 10 cm and a radius of 5 cm?



5. Have students solve a variety of measurement problems involving ratios. For example:

   - How does the volume of a pyramid compare to the volume of another pyramid that has the same base but twice the height?
   - How does the volume of a pyramid compare to that of a pyramid that has the same height but twice the base area?

F. Financial Literacy

Overall Expectation

By the end of this course, students will:

F1. Financial Decisions: demonstrate the knowledge and skills needed to make informed financial decisions

Overall expectation

F1. Financial Decisions

demonstrate the knowledge and skills needed to make informed financial decisions

Specific expectations

By the end of this course, students will:

F1.1 Financial Decisions

identify a past or current financial situation and explain how it can inform financial decisions, by applying an understanding of the context of the situation and related mathematical knowledge

Teacher supports

Examples

- **financial situations that students may identify:**
  - systemic, current, or historical events and issues, or economic shifts, that happen at a global, national, local, or individual level
- **considerations for understanding the context of the situation to inform financial decisions:**
  - What factors contribute to a financial situation?
  - Who does the financial situation affect?
  - What factors contribute to a financial decision?
  - Who has the ability to make decisions as a result of the situation?
  - Who is affected by the decisions?
  - What are the impacts of the decisions?
  - What aspect of mathematics is helpful in order to understand the situation and the decisions that might be made?
- **related mathematical knowledge:**
  - proportional reasoning
  - operation sense
  - linear and non-linear relations

   - graphical analysis
   - data analysis

Instructional Tips

Teachers can:

- facilitate students' sharing of the financial situations they are exploring, in a safe, supportive, and inclusive environment that is free of judgement and respects family, community, and cultural expectations, practices, and perspectives;
- be flexible and create space for discussions based on authentic and relevant opportunities as they emerge from current social issues;
- ensure that all examples of financial situations discussed, although authentic and connected to students' realities, are generalized or fictionalized and free of judgement;
- provide students with access to a variety of print and digital resources to research financial situations;
- support students in making connections between the financial situations they are exploring and mathematical concepts they are learning throughout the course.

Teacher Prompts

- What are some past or current financial situations, events, or issues that have had an impact on your local communities, country, or the world at large?
- What factors contributed to this financial situation and who does/did it affect?
- What are some of the impacts of this situation and the decisions around it?
- What sources could you use to gather more information about the context of the situation?
- What mathematics could you use to better understand this situation and its impact?
- What mathematics would you need to learn more about to better understand this situation and its impact?

Sample Tasks

Have students brainstorm financial situations and then, in groups or with a partner, choose a situation to discuss more thoroughly. Ask them to identify what mathematical concepts would be helpful in understanding the situation and the decisions that might be made. (See the Examples for questions that students might consider in their discussion.)

F1.2 Financial Decisions

identify financial situations that involve appreciation and depreciation, and use associated graphs to answer related questions

Teacher supports

Examples

- **financial situations that involve appreciation or depreciation of assets over time:**
  - the purchase, ownership, or sale of:
    - collectibles
    - electronic goods
    - vehicles
    - real estate
    - stocks and other investments

Instructional Tips

Teachers can:

- compile a list of relevant student-generated examples through class discussions about goods or assets that appreciate and depreciate;
- ensure that the examples are authentic and relevant to students' lives, by drawing from topics generated by students;
- provide opportunities for students to:
  - interpret graphs that show appreciation or depreciation, including identifying possible causes for the trends, making predictions about future trends, and discussing possible implications;
  - make connections to characteristics of linear and non-linear relationships, including exponential relationships, such as increasing and decreasing trends and rates of change when interpreting graphs (see C3.1).

Teacher Prompts

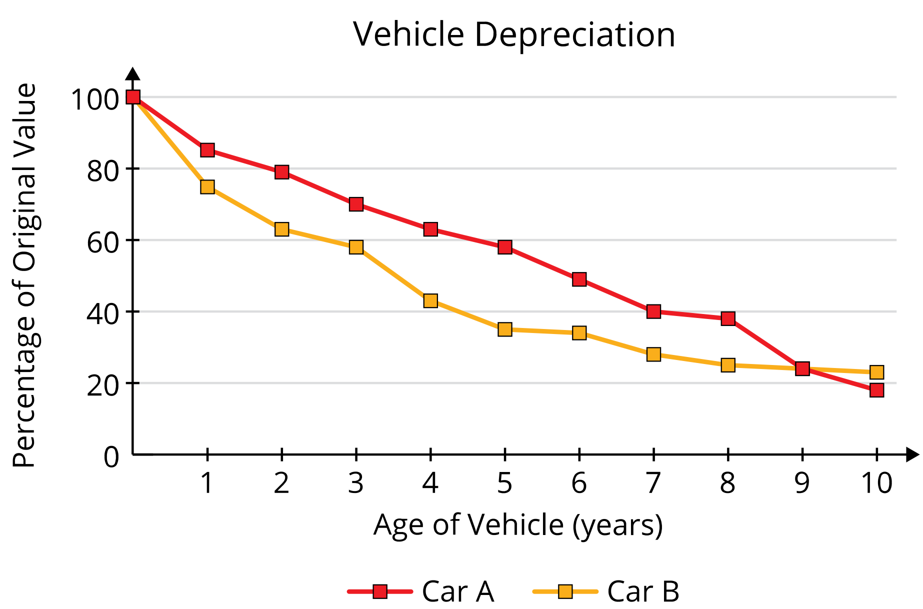
- What are some examples of assets that appreciate or depreciate?
- What might affect the rate at which an asset appreciates or depreciates?
- What causes some assets to appreciate while others depreciate?
- Given a graph showing appreciation and depreciation rates, identify the following:
  - What variables are involved? What is being measured on the vertical axis? On the horizontal axis?
  - During what time period(s) is the value increasing or decreasing quickly?

- At what point has the value of the item dropped to half the original value or increased to double the original value?
  - What might have caused the change in value at a given time and how might this affect the people involved?
  - How does the depreciation or appreciation of [a given item] compare to the depreciation or appreciation of [another given item]?
- Why do you think a business or non-profit organization has to take into account the depreciation of its assets when it does year-end accounting?

Sample Tasks

1. Have students brainstorm examples of assets that appreciate or depreciate, including those that might experience a short-term appreciation due to a current trend (e.g., trading cards, trends started on social media). Show students graphs or provide them with data to graph depicting the appreciation and depreciation of assets identified during the brainstorming session and have them identify what they notice and what questions they might still have about each of the graphs.

2. Show students a graph comparing the depreciation of two different vehicles, such as the graph below.



Ask students:

- What variables are represented on each of the axes?
- Why might one vehicle depreciate at a different rate than the other?

- When (after how many years) is each vehicle worth less than half its original value?
- Why might it be helpful to understand how vehicles depreciate?
- Could a vehicle ever appreciate in value?

Have them choose a vehicle they are curious about, look up the depreciation rate for that vehicle, and then compare the rate with that of a vehicle chosen by a classmate.

3. Have students investigate an item they think has appreciated or depreciated over time. Ask: Did the value of this item appreciate or depreciate as you expected? Have students find or create a graph to show how the value has changed.

4. Have students write code to analyse the value of an item over time for different rates of appreciation. The following is an example of pseudocode that determines the value of an item that appreciates at a given rate for each period of time and graphs the results. Have students alter the code for a situation involving depreciation, and have them consider which items appreciate or depreciate in value.

```
costOfItem = 0.00
timePeriod = 0
appreciationRate = 0.00
output “Enter the cost of an item.”
store user input as costOfItem
plot point (timePeriod, costOfItem)
output “Enter the rate of appreciation as a percentage.”
store user input as appreciationRate
while appreciationRate < 1
    output “Enter the rate of appreciation as a number
    greater than 1.”
    store user input as appreciationRate
repeat until timePeriod = 10
    timePeriod = timePeriod + 1
    costOfItem = costOfItem * (1 + appreciationRate/100)
    plot point (timePeriod, costOfItem)
```

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.

F1.3 Financial Decisions

compare the effects that different interest rates, lengths of borrowing time, ways in which interest is calculated, and amounts of down payments have on the overall costs associated with purchasing goods or services, using appropriate tools

Teacher supports

Examples

- **ways in which interest is calculated:**
  - simple interest
  - compound interest
  - different compounding periods, such as weekly, monthly, biannually, or annually
- **tools:**
  - spreadsheets
  - graphing tools
  - coding
  - online financial calculators

Instructional Tips

Teachers can:

- facilitate a class discussion, using authentic and accessible contexts, about how changing different variables might affect overall costs of a purchase, then have students complete a numerical analysis;
- incorporate contexts from previous grades such as loans, credit cards, or lines of credit;
- create opportunities for students to engage in estimation and make conjectures before calculating the actual values, then have them compare their estimations with the results and reflect on the reasons for any differences;
- support students in using available technology and coding tools to examine how changing one variable might affect overall costs so that the focus of the learning can be on the effects of changing variables rather than on making complex calculations.

Teacher Prompts

- What effect will increasing or decreasing each of the following have on the overall cost (of goods or services)?
    - the interest rate
    - the length of the borrowing time
    - the frequency at which interest is calculated
    - the amount of a down payment
- Why does increasing the length of the borrowing time increase the total cost of an item?
- How does a lower interest rate compounded monthly compare to a higher interest rate compounded annually?

Sample Tasks

1. Provide students with a chart similar to the one below. Have them indicate whether certain changes in variables would increase or decrease the overall cost of an item and explain why. Encourage students to use an online or handheld financial calculator to test their ideas.

Variable	Change in variable	Effect on cost (circle one)	Explanation
interest rate	increase	Increase / decrease / remain the same	
loan term length	increase	Increase / decrease / remain the same	
amount of down payment	increase	Increase / decrease / remain the same	

2. Ask students to choose an item they might want to purchase sometime in the future (e.g., computer, gaming system, used vehicle, bicycle). Have them use a financial calculator to explore the effects of changing the conditions of a loan to purchase the item, such as the interest rate or term length.

3. Provide students with a sequence of code to calculate the total cost and interest paid on a loan that has interest compounded monthly. Have students alter the code to explore how changing the compounding period (e.g., compounding annually, semi-annually, weekly) would change the total cost and interest for that loan.

amountOfLoan = 0.00
borrowingTime = 0
interestRate = 0.00
output "Enter the amount of the loan."
store user input as **amountOfLoan**
output "Enter the borrowing time for the loan in years."
store user input as **borrowingTime**
output "Enter the annual interest rate as a percentage."
store user input as **interestRate**
interestRate = **interestRate**/100
compoundPeriods = borrowingTime * 12
totalCost = **amountOfLoan** * (1 + **interestRate**/12)^**compoundPeriods**
totalInterest = **totalCost** - **amountOfLoan**
output "The total amount paid at the end of the loan would be" **totalCost**
output "The total interest paid would be" **totalInterest**

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.

F1.4 Financial Decisions

modify budgets displayed in various ways to reflect specific changes in circumstances, and provide a rationale for the modifications

Teacher supports

Examples

- **ways of displaying a budget:**
    - budget template:

Monthly Budget

Income

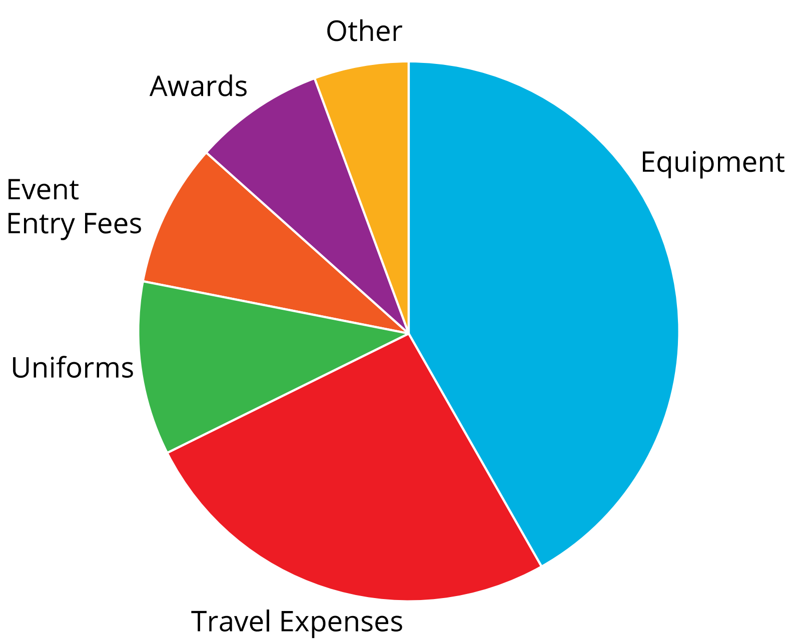
Monthly Income	
Other Income	
Total Income	

Expenses

Mortgage or Rent	
Electricity	
Gas	
Water	
Transportation Costs	
Insurance	
Food	
Phone	
Taxes	
Savings	
Other	
Total Expenses	

- circle graph:

School Team Budget



- digital spreadsheet or software

- **changes in circumstances:**
  - a change in income or revenue
  - a change in the cost of utilities
  - a change in transportation costs
  - a change in operational costs
- **modifying budgets:**
  - adjusting income or expense categories in a budget to create a balanced budget, whether for an individual, a family, an organization, a business, or a government

Instructional Tips

Teachers can:

- share examples of budgets represented in various ways;
- provide students with opportunities to work with a variety of budgets, such as those for an individual, a family, a business, an organization, a community-based non-profit organization, or a government (municipal, provincial, or federal);
- provide students with opportunities to discuss and explore various categories that might be included in a budget;
- ensure that all examples provided or discussed, although authentic and connected to students' realities, are hypothetical and free of judgement;
- create opportunities for students to work in pairs or small groups to brainstorm how a change in circumstances might affect a budget;
- have students modify budgets and justify their modifications through discussion or in writing.

Teacher Prompts

- What are some of the categories you might find on a family budget? a school team's or club's budget? a small business's budget?
- In general, which categories of expenses and revenue on a budget are easier to change? Which are more difficult to change?
- What are some circumstances that would result in the need for a change in a budget?

Sample tasks

1. Provide students, working in small groups, with a sample balanced budget for a family that includes realistic values for the local community. Give each group a realistic circumstance that would require a change or changes in the budget. Have the group work collaboratively to adjust the budget to keep it balanced and then share their changes and rationale with the class. *Note:* Ensure that all

examples provided or discussed, although authentic and connected to students' lives, are hypothetical and free of judgement.

2. Show students the budget for a division of the local municipal government (e.g., Parks and Recreation). Pose a scenario that is relevant to the current local situation (e.g., community members would like an outdoor skating rink) and have students discuss how the budget could be modified based on this scenario.

3. Provide students with a budget for a school trip in the form of a circle graph. Have them consider how the budget would need to be changed for a variety of circumstances, such as:

   - an increase in transportation costs
   - a decrease in the number of people coming on the trip
   - a donation from a community organization to help fund the trip

Glossary

The definitions provided in this glossary are specific to the curriculum context in which the terms are used.

algebraic expression

A collection of one or more terms involving variables, numbers, and operations. For example, the algebraic expression 5m has one term, and $6x^2 + xy - 8$ has three terms. *See also* **term, variable**.

appreciation

An increase in value of an asset or a currency.

assumption

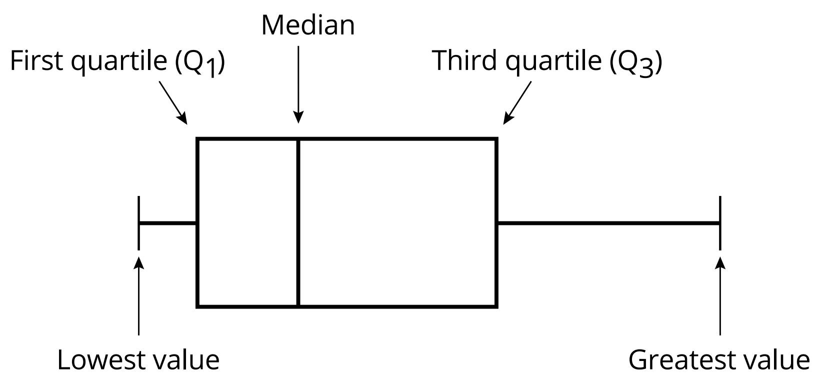
A premise that a person believes to be true.

base

A factor or the value in a power that is being repeatedly multiplied. For example, in the power 3^5, 3 is the base. *See also* **exponent, power**.

box plot

A graphic representation of the spread of a data set. A rectangle (box) shows the spread of the central half of the distribution, with the first quartile on the left edge, the third quartile on the right edge, and the median as a line within the box. Lines (whiskers) extend from the sides of the box to the lowest and highest values that are not outliers. Potential outliers are marked with a symbol beyond the whiskers. Also known as *box-and-whisker plot*. *See also* **median**, **quartile values**.

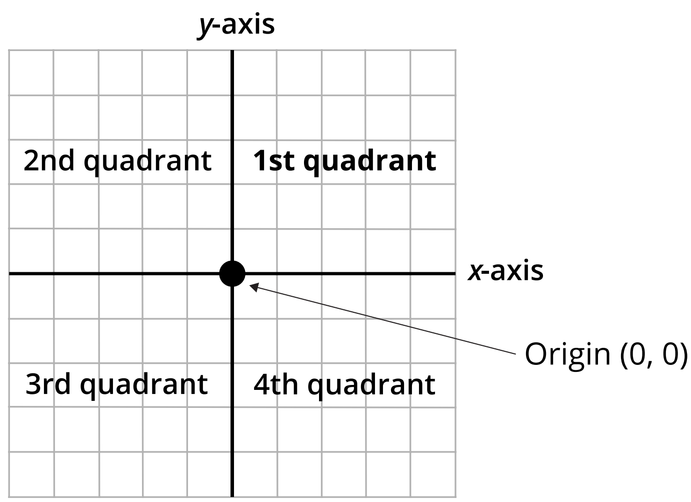


budget

An estimate or plan to manage income and expenses over a set period; for example, many people have a weekly or monthly budget. *See also* **expenses, income**.

Cartesian plane

A two-dimensional coordinate system divided into quadrants by a horizontal axis (*x*-axis) and a vertical axis (*y*-axis) intersecting at a point called the *origin*. The location of any point (*x*, *y*) on the *x*-*y* plane is described relative to the origin (0, 0). For example, the point (3, 4) is located within the first quadrant, 3 units to the right of the *y*-axis and 4 units above the *x*-axis.



characteristic

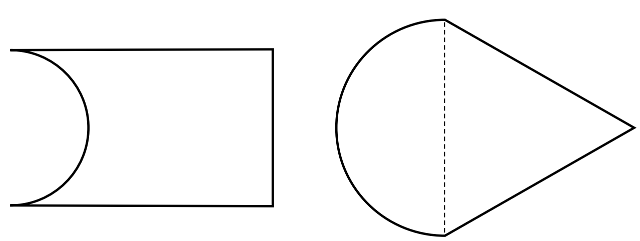
A distinguishing trait or quality. For example, any line defined by x = k has the characteristic of being a vertical line parallel to the *y*-axis on the *x*-*y* plane.

coding

The process of writing computer programming instructions.

composite shape

A shape composed of two or more basic shapes.

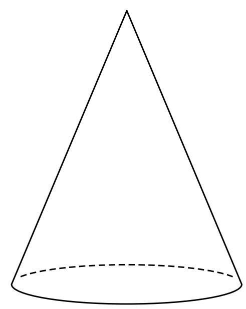


conceptual understanding

A deep understanding of mathematical ideas that goes beyond isolated facts and procedures to recognizing the connections between and usefulness of mathematical ideas in various contexts. For example, having a conceptual understanding of place value helps in understanding the various procedures involved when doing operations such as multiplying multi-digit or decimal numbers.

cone

A three-dimensional object with a circular base and a curved surface that tapers proportionally to an apex. *See also* **three-dimensional object**.



constant

A part of an algebraic expression that does not change. For example, in the expression $x + y = \mathrm{k}$, k represents a constant and x and y are the variables. When k is equal to 1, 1 will remain the same, and the values of x and y can vary as long as they have a sum of 1. *See also* **algebraic expression**, **variable**.

constraint

A restriction placed on the parameters upon which a subroutine or program works or in a loop to define the scope of the problem.

coordinate system

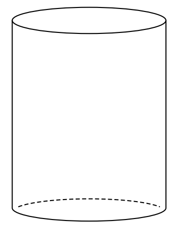
A system used to specify location on a grid. The Cartesian plane is an example of a coordinate system. *See also* **Cartesian plane**.

correlation

A measure of the strength and the direction of the linear relationship between two variables. The correlation coefficient is the numerical measure of a correlation, and the closer the correlation coefficient is to 1 or −1, the stronger the linear relationship is between the two variables. A negative correlation indicates that as one variable increases in value, the other variable decreases in value.

cylinder

A three-dimensional object with two congruent, parallel faces and one curved surface. All cross-sections parallel to the base are identical. For example, a right circular cylinder (shown below) has two circular faces and a curved surface that is perpendicular to the base. *See also* **three-dimensional object**.



density

The concept that between any given two real numbers, there will always be another real number. Thus, there are infinitely many real numbers between any two real numbers. *See also* **infinity, real number**.

dependent variable

A variable whose value depends on the value of another variable for a particular situation. For example, in the expression $d = vt$, distance (d) depends on velocity (v) and time (t). In graphing, the dependent variable is usually represented on the vertical axis of a Cartesian plane. *See also* **independent variable, variable**.

depreciation

A decrease in value of an asset or a currency.

dispersion

The spread of values in a data set.

earning

Obtaining money in return for labour or services. *See also* **income**.

equation

A mathematical statement that has equivalent expressions on either side of an equal sign.

evaluate

To determine a value for an expression.

expenses

Things that one spends money on; for example, most adults' expenses include food, shelter, utilities, and entertainment.

exponent

The value in a power defining the operation on the base. For example, the exponent 3, in the power 5^3, defines that three 5s are multiplied together. *See also* **base, power**.

exponential decay

A decrease in a quantity over time, in which the quantity is diminished by a consistent fraction or percentage over a period of time. For example, when quantity *n* decreases by half, the exponential decay can be written as $n \times 0.5$, $n \times 0.5 \times 0.5$, $n \times 0.5 \times 0.5 \times 0.5$, $n \times 0.5 \times 0.5 \times 0.5 \times 0.5$, ... to show the initial value of *n* being halved in each time period. *See also* **exponential growth**.

exponential growth

An increase in a quantity over time, in which the quantity is increased by a consistent multiple over a period of time. For example, when quantity *n* doubles, the exponential growth can be written as $n \times 2$, $n \times 2 \times 2$, $n \times 2 \times 2 \times 2$, $n \times 2 \times 2 \times 2 \times 2$, ... to show the initial value of *n* being doubled in each time period. *See also* **exponential decay**.

expression

A numeric or algebraic representation of a quantity. An expression may include numbers, variables, and operations; for example, $3 + 7$, $2x - 1$. *See also* **algebraic expression**.

fraction

A number in the form $\frac{a}{b}$, in which the numerator *a* and the denominator *b* are integers and $b \neq 0$. For example, $\frac{1}{2}$, $\frac{17}{10}$, $\frac{3}{3}$, and $\frac{-1}{4}$ are all fractions. *See also* **rational number**.

generalize

To make a statement that is consistent with all specific instances.

geometric property

An attribute that remains the same for a class of objects or shapes. For example, an attribute for any parallelogram is that its opposite sides are of equal length.

growing pattern

A pattern that involves an increase from term to term. A growing pattern that has a constant increase from term to term, such as 3, 7, 11, 15, … , is an example of a linear growing pattern. A growing pattern that does not have a constant increase from term to term, such as 3, 6, 12, 21, … is an example of a non-linear growing pattern. *See also* **shrinking pattern**.

income

Money that an individual receives in exchange for work or from investments. *See also* **earning**.

independent variable

A variable for which values are not dependent on the values of other variables. For example, in the expression $b = 2 + c$, c is the independent variable because it does not depend on another variable for its value. In graphing, the independent variable is usually represented on the horizontal axis of a Cartesian plane. *See also* **dependent variable**.

inequality

The relationship between two expressions or values that are not equal, indicating with a sign whether one is less than (<), greater than (>), or not equal to (≠) another. An inequality can include an equal component such as less than or equal to (≤) and greater than or equal to (≥). For example, $a < b$ means a is less than b and $a \geq b$ means a is greater than or equal to b. *See also* **equation**.

infinity

The state of having no end or limit. For example, the set of even numbers or the set of rational numbers cannot be counted. A pattern or expression is said to approach infinity if the value can always be made larger than any given value.

initial value

The value of the dependent variable when the independent variable is equal to zero. For example, in the expression $c = 50 + 2b$, when $b = 0$, $c = 50$. *See also* **dependent variable**, **independent variable, rate of change**.

integer

Any one of the numbers …, –4, –3, –2, –1, 0, +1, +2, +3, +4, ... Integers are the entire set of whole numbers and their opposites (negative numbers).

irrational number

A real number that is not a rational number. Thus, it is a number that cannot be represented as a fraction, and when expressed as a decimal it does not terminate or repeat; for example, $\sqrt{5}$, pi.

knowledge systems

Knowledge systems are developed over time by specific groups of people in particular locations around the world and passed on from generation to generation. A range of knowledge systems, including the rich diversity of Indigenous knowledge systems, shares related world views on key core values, beliefs, and practices and reflects the depth of locally held knowledge that is often rooted in a culture and place.

Understanding various knowledge systems is necessary for more than culturally relevant or responsive education because for some cultures and communities, such as diverse Indigenous peoples from around the world, the knowledge is connected to the land and part of a collective history and contemporary knowledge and perspectives.

limit

The long-term behaviour of a pattern or function, or the result as the number of terms increases. For example, the limit of the value of $\frac{a}{b}$ (for positive values of *a* and *b*) as *b* approaches 0 is infinity.

linear relation

A relation between two variables that appears as a straight line when graphed on a coordinate system. May also be referred to as a *linear function*.

mathematical model

1) A representation of a mathematical idea. For example, a number line is a model of a mathematical idea, as it shows the order and magnitude of numbers.
2) A mathematical solution to a complex real-life situation, created through the mathematical modelling process.

mathematical modelling process

An iterative and interconnected process of using mathematics to represent, analyse, make predictions, and provide insight into real-life situations. This process involves four components: understanding the problem, analysing the situation, creating a mathematical model, and analysing and assessing the model.

mathematical processes, the

The set of interconnected actions involved in doing mathematics. In the Ontario mathematics curriculum, the seven mathematical processes are problem solving, reasoning and proving, representing, reflecting, selecting tools and strategies, connecting, and communicating.

mean

One of the measures of central tendency. The mean represents the value that each piece of data would have if the data were evenly distributed. It can be calculated by adding up all the numbers and then dividing the result by the number of numbers in the set. For example, the mean of 10, 20, and 60 is (10 + 20 + 60) ÷ 3 = 30. Also called *average*. *See also* **measures of central tendency, median, mode**.

measurement system

A collection of measurement units and rules that define these units' relationships to each other.

measures of central tendency

A set of measures that represent the approximate centre of a set of data. Mean, median, and mode are all measures of central tendency. *See also* **mean**, **median**, **mode**.

median

One of the measures of central tendency. The median is the middle value of a list of numbers sorted in ascending or descending order. For example, 14 is the median for the set of numbers 7, 9, 14, 21, 39. If there is an even number of data values, then the median is the average of the two middle values. *See also* **mean**, **measures of central tendency**, **mode**.

mixed number

A number that is composed of an integer and a fraction; for example, $-8\frac{1}{4}$.

mode

One of the measures of central tendency. The mode is the category with the greatest frequency, or the number that appears the most in a set of data. For example, in a set of data with the values 3, 5, 6, 5, 6, 5, 4, 5, the mode is 5. *See also* **mean**, **measures of central tendency, median**.

model

Representation of a problem, situation, or system using mathematical concepts.

non-linear relation

A relation between two variables that does not appear as a straight line when graphed on a coordinate system.

number system

A way in which number relationships are defined. For example, the base ten number system includes the digits 0 to 9, and the relationship of one place value to the next is a multiple of 10.

parameter

A special type of variable used in the definition of a subprogram, which defines the values that are inputted when the subprogram is executed.

percent

A ratio with a second term of 100. A percent is expressed using the symbol %. For example, 30% means 30 out of 100. A percent can be represented by a fraction with a denominator of 100; for example, 30% = $\frac{30}{100}$.

point of intersection

The point at which two or more lines or curves cross. Two lines may have one point of intersection, no points of intersection, or an infinite number of points that intersect.

population

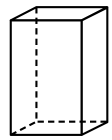
The total number of individuals or items under consideration in a surveying or sampling activity.

power

A number written in exponential form. For example, in the power 2^5, the base is 2 and the exponent is 5. *See also* **base, exponent**.

prism

A three-dimensional object with two parallel and congruent faces. A prism is named by the shape of its bases, for example, rectangle-based prism, triangle-based prism. *See also* **three-dimensional object**.



Rectangle-Based Prisms

probability

The likelihood that an event will occur. Probability is often represented as a percentage between 0 and 100 or as a decimal between 0 and 1.

procedural fluency

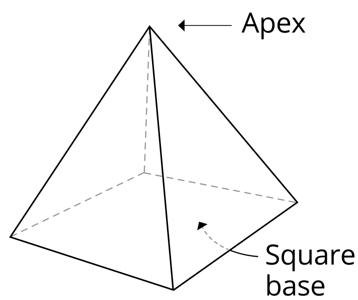
The ability to use procedures in an accurate, efficient, and flexible way to solve problems.

proportion

The equivalence of two or more ratios; for example, 3:m:2 = n:8:10. *See also* **ratio**.

pyramid

A three-dimensional object whose base is a polygon and whose other faces are triangles that meet at a common vertex called the *apex*. A pyramid is named by the shape of its base, for example, square-based pyramid, triangle-based pyramid.



quantitative data

Data that is numerical and acquired through counting or measuring; for example, number of sides of a three-dimensional object or amount of rainfall in a season.

quartile values

Values that divide a sequenced data set into four parts, each of which represents 25% of the data falling within that range. For example, 25% of the data is below the point that defines the first quartile, which is the middle number between the smallest number in the data set and the median. *See also* **box plot**.

quotient

The result of a division.

rate

A comparison, or a type of ratio, of two measurements with different units; for example, 100 km/h, 10 kg/m^3, 20 L/100 km. *See also* **ratio**.

rate of change

The change in one variable relative to the change in another. The slope of a line represents a constant rate of change. *See also* **slope**.

ratio

A comparison of quantities with the same units. A ratio can be expressed in ratio form or in fraction form, for example, 3:4 or $\frac{3}{4}$.

rational number

A number that can be expressed as a quotient of two integers where the divisor is not zero. It can also be represented as a decimal number that either repeats or terminates. For example, $\frac{1}{3}$ or 0.3333..., $\frac{17}{10}$ or 1.7, $\frac{3}{3}$ or 1, and $\frac{-1}{4}$ or −0.25 are rational numbers. *See also* **fraction, quotient**.

real number

A rational or irrational number. *See also* **rational number, irrational number**.

reflection

A transformation that flips points over a line, such as the *x*-axis or *y*-axis, such that the reflected point and the original point are the same distance perpendicularly from the line of reflection.

regression

A statistical method for determining the relationship between the dependent variable and the independent variable for a set of data.

relation

An identified relationship between two variables that may be expressed as a table of values, a graph, or an equation.

rotation

A transformation that turns a set of points about a fixed point, such as the origin on a Cartesian plane, usually involving rotations that are multiples of 90 degrees. *See also* **transformation**.

sample

A subset of a population. *See also* **population, subset**.

scatter plot

A graph designed to show a relationship between corresponding numbers from two sets of data measurements associated with a single object or event; for example, a graph of data about students' marks and the corresponding amounts of study time. Drawing a scatter plot involves plotting ordered pairs on a coordinate grid.

scientific notation

A way of expressing a very large or very small number in terms of a decimal number between 1 and 10 multiplied by a power of 10. For example, 690 890 000 000 is 6.9089×10^{11} in scientific notation, and 0.000279 is 2.79×10^{-4}. *See also* **power**.

set

A collection of elements that fulfil specific criteria.

shrinking pattern

A pattern that involves a decrease from term to term. A shrinking pattern such as −3, −7, −11, −15 is linear since there is a constant decrease of 4 from term to term. This type of shrinking pattern is also known as a decreasing pattern. A shrinking pattern such as 40, 20, 10, 5, 2.5 is an example of a shrinking pattern that is non-linear, since the decrease from term to term is not constant. *See also* **growing pattern**.

simplify

To create an equivalent fraction by dividing the numerator and denominator by their greatest common factor, or by performing operations to create fewer terms in an algebraic expression.

slope

A measure of the steepness of a line, calculated as the rate of the rise (vertical change between two points) to the run (horizontal change between the same two points).

source

A place where data is obtained. Types of sources include *original (primary) sources*, such as observations, conversations, and measurements, and *secondary sources*, such as magazines, newspapers, government documents, and databases.

statistically analyse

To analyse a data set according to its measures of central tendency or its dispersion. *See also* **dispersion, measures of central tendency**.

subset

A smaller set within a set. For example, the set of whole numbers is a subset of the set of integers. *See also* **set**.

symmetry

The geometric property of being balanced about a point, a line, or a plane.

term

A single number, variable, or combination of numbers and variables involving no addition or subtraction.

three-dimensional object

An object that has the dimensions of length, width, and depth.

transformation

A change in a set of points that results in a different position on a coordinate system. Transformations include translations, reflections, rotations, and dilations.

translation

A transformation that moves every point in a set the same distance, in the same direction.

unit

A quantity used as a standard of measurement.

unit fraction

Any fraction that has a numerator of 1, for example, $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$. Every fraction can be decomposed into unit fractions. For example, $\frac{3}{4}$ is 3 one-fourth units, or $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

variable

A symbol, letter, or words representing a value that can vary depending on the context. For example, in an equation $x + y = 3$, if the variable x changes, the value of variable y will change accordingly. In coding, words are often used instead of a letter, such as FirstNumber + SecondNumber = 3. In coding, a variable is a temporary storage location for data such as a numerical value or a series of characters, and the values stored in that location vary depending on the commands given by the program. In a data set, a variable such as height may take on a different value for each member of a population.

x-intercept

The value of x for a point (x, y) on the x-axis when y is zero.

y-intercept

The value of y for a point (x, y) on the y-axis when x is zero.